DOCUMENT RESUME

ED 417 938 SE 061 190

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TITLE Development of Course Materials To Integrate Precalculus Review with the First Course in Calculus.
INSTITUTION Moravian Coll., Bethlehem, PA.
SPONS AGENCY Fund for the Improvement of Postsecondary Education (ED), Washington, DC.
PUB DATE 1996-00-00
NOTE 94p.
CONTRACT P116B12036
PUB TYPE Reports - Descriptive (141)
EDRS PRICE MF01/PC04 Plus Postage.
DESCRIPTORS Calculus; Course Descriptions; *Curriculum Enrichment; Higher Education; *Instructional Materials; Mathematics Education; *Remedial Instruction; *Textbook Preparation
IDENTIFIERS *Precalculus

ABSTRACT

This document describes a two-year project designed to prepare, desk-top publish, and class-test supplemental materials geared specifically toward a slower-paced, one-year Calculus I course that integrates the review of precalculus topics as they are needed. This course replaces the traditional two-semester Precalculus-Calculus I sequence for underprepared students. This report includes an executive summary involving project overview, statement of purpose, background and origins, project description, evaluation/project results and conclusions. A 1992 draft of the text: "A Companion To Calculus"; an evaluation report of the two-year project; announcements and program materials from the dissemination conference, and two papers related to this project are contained in appendices. (ASK)

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ED 417 938

Final Report, FIPSE project

*Development of course materials to integrate precalculus review with the first course in Calculus*

Grantee Organization:

Moravian College
Mathematics Department
1200 Main Street
Bethlehem, PA 18018-6650

Grant Number:

P116B12036

Project Dates:

Starting Date: August 20, 1991
Ending Date: August 19, 1993
Number of Months: 24

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| Grant Award: | Year 1 | $54,625 |
| | Year 1 supplement | $30,350 |
| | Year 2 | $ 7,923 |
| | Total | $92,898 |

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Summary Paragraph

Our 2-year FIPSE-funded project has been to prepare, desk-top publish, and class-test supplemental materials especially designed for a slower-paced one year Calculus I course that integrates the review of precalculus topics *as they are needed*. This course replaces the traditional 2-semester Precalculus-Calculus I sequence for underprepared students. An extensive comparative evaluation of student performance and attitudes was carried out during the two-year period in all sections of the course at Moravian College. Dissemination activities included presentations at national conferences, published articles about the project, and a dissemination conference held at Moravian College in June 1993. Our text, entitled "A Companion to Calculus," is intended to be used with any first year calculus text. It was sought by several publishers and a preliminary version will be published in January 1994 by Brooks/Cole. The current draft of the text is being used at 9 institutions during the 1993-1994 academic year.

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Title of text: *A Companion to Calculus*, to be published by Brooks/Cole, Pacific Grove, CA.

Published reports:

"Integrating Precalculus Review with the First Course in Calculus," Alicia Sevilla and Kay Somers, PRIMUS, vol. III, no. 1, 1993, pp. 35–41.

"A Report on a Project to Develop Course Materials to Integrate Precalculus Review with the First Course in Calculus," Alicia Sevilla, Kay Somers, and Doris Schattschneider, Proceedings of the Conference on Calculus and Precalculus Reform (Monticello, Illinois, April 1993), to be published by the Mathematical Association of America, Fall 1993.

## Executive Summary

Project: *Development of course materials to integrate precalculus review with the first course in Calculus*

Grantee: Moravian College, 1200 Main Street, Bethlehem, PA 18018-6650

Project Director: Doris Schattschneider, telephone (215)-861-1373

### Project Overview

Our project began as a response to a serious problem: roughly half of the students required to take calculus in college are inadequately prepared. Until 1988, Moravian College taught a one-term Precalculus course to these students. Attrition in the course was high, morale was low, and students did not retain material needed the next semester in calculus.

In 1988, a new one-year course "Calculus I With Review" replaced the Precalculus-Calculus I sequence at Moravian. The course integrates the review of precalculus topics *as they are needed* within the first calculus course. Although many Precalculus books are available, there are no published materials that are designed to be used in such an integrated course. Our 2-year project was to prepare, desk-top publish, and class-test supplemental materials especially designed to integrate the review of algebra skills, functions, graphing, and problem-solving techniques in the beginning calculus course.

Throughout the academic year 1991-1992, three faculty members from Moravian College and a faculty member from Northampton Community College developed the first draft of the supplemental materials. Entitled "A Companion to Calculus," they were used in all sections of Calculus I with Review at Moravian College. During the summer 1992, the "Companion" was revised and it was used at Moravian College in 1992-1993 and served as a supplement in a one-semester Calculus I course at Northampton Community College.

Evaluation of the project was ongoing throughout the 2-year period: records on student enrollment and completion of the course as well as the former sequence it replaced; comparison of individual student performance on pretests and post-tests; comparison of Calculus I with Review student performance on selected exam questions with those in the "regular" one-semester course; attitude surveys of those in the integrated course; evaluation of the materials by students and instructors.

Dissemination was also a major part of the project. Presentations about the project were made at the 1992 Conference on the Teaching of Calculus held at Harvard University, the 1993 national MAA-AMS meetings in San Antonio, and an NSF-sponsored conference on Precalculus and Calculus Reform held at the University of Illinois Allerton Conference Center in April 1993. A dissemination conference was held at Moravian College on June 18–19, 1993, at which sixty teachers from over 40 institutions of higher learning participated.

As a result of the dissemination efforts, the second draft version of the "Companion" is being used at 9 colleges and universities during the 1993-94 academic year. In September, 1993, a publishing contract with Brooks/Cole was signed.

### Purpose

Our goal was to produce materials to use in a one-year course that integrates the review of needed precalculus concepts and skills with the introduction of calculus. The larger purpose was to provide access to calculus to those who are underprepared, and to try to address the problem of poor skills, attitude, and morale that is pervasive among such students. We also wished to make our approach known to other institutions of higher learning.

Although our evaluation indicates that we were successful in producing materials that most students found readable and helpful, the materials alone cannot address the problem of turning around long-standing attitudes toward mathematics and study habits that lead to failure.

### Background and Origins

At Moravian College, 60% of the freshman class intend to pursue a course of study that requires calculus, yet roughly half of these students are inadequately prepared to take calculus. There are few institutions of higher learning in the United States that do not share the problem.

The standard response of most institutions is to require such underprepared students to first complete a one-semester course called "Precalculus." Until 1988, Moravian College followed this standard pattern . The morale of both students and teachers in the Precalculus course was low; the attrition in the course was fairly high; the retention of material needed for Calculus I the next semester seemed extremely low.

In 1987, the Mathematics Department at Moravian proposed to the faculty that the Precalculus course be dropped and a new one-year course, Calculus I with Review, replace the Precalculus-Calculus I sequence for those students who were not prepared to begin with Calculus I. Although there was spirited discussion about the proposal, the faculty approved the change. Begun in 1988, the course integrates the review of precalculus topics *as they are needed* within the first calculus course. The same calculus text and calculus syllabus is used in the one-semester Calculus I course and in the slower-paced one-year course. In the first two years the integrated course Calculus I With Review was offered, there was an observed improvement in morale, level of understanding, and numbers completing the course.

The greatest difficulty of teaching the integrated course was the lack of published material to provide the needed background and review, but in a calculus setting. Precalculus texts are not designed to be used as part of a calculus course. In 1990 we wrote the grant proposal to FIPSE to support the preparation and production of the supplemental materials, a careful evaluation of the course and use of the materials, and dissemination activities for the project.

In writing our proposal, we sought and received promises of support from Moravian College and Northampton Community College and the Lehigh Valley Association of Independent Colleges. We also sought and received letters of interest in the proposed materials from two publishers.

**Project Description**

Our funding began on August 20, 1991. During the last two weeks of August 1991, project team members met for several hours daily and outlined the supplemental material to be written for the year-long course, as well as prepared and duplicated materials for the first month of the course. A working pattern was established that was followed throughout the year: for each calculus topic, the group of four discussed the pitfalls that were typical for students with weak background, identified possible ways to address these items, then assigned a team member write a chapter. These first drafts were then heavily critiqued in open discussion by the team and revised.

Information in the materials is presented through four different modes: words, figures (graphs and diagrams), symbols (algebraic formulations), and numerical data. The text contains many examples and exercises, and encourages the use of technology, though is not dependent on this technology. Entitled "A Companion to Calculus" it was produced throughout the year, and class-tested in all sections of Moravian's Calculus I with Review. (See Appendix I for the text.)

During the summer of 1992 the "Companion" was revised to produce a 20-chapter volume with fairly uniform appearance and style. During the 1992-1993 academic year, this revised draft was used in all sections of Moravian's Calculus I with Review. In addition, Dennis Ebersole used it as a required supplement to a one-semester Calculus I course at Northampton Community College.

Dissemination was also a primary focus of the project during the 1992-1993 year. Information about our project was first disseminated through the project director's description prepared for the FIPSE Project Directors' meeting in Fall 1991. The following June, Kay Somers and Alicia Sevilla made a presentation at the Conference on the Teaching of Calculus held at Harvard University (see Appendix 4). During the second year of the project, more than 40 copies of the "Companion" were sent to those who requested it. In January 1993, at the national MAA-AMS meetings in San Antonio, Doris Schattschneider and Dennis Ebersole participated in a 4-hour poster session devoted to calculus reform projects. In April 1993, Doris Schattschneider was a panelist at an NSF-sponsored conference on Precalculus and Calculus Reform held at the University of Illinois Allerton Conference Center (see Appendix 5). On June 18-19, 1993, a dissemination conference "The Integration of Precalculus with Calculus" was held at Moravian College. It featured an address by Dr. Susan Forman of the Mathematical Sciences Education Board, a panel of four teachers from institutions with integrated courses, and 12 contributed papers. (See Appendix 3.) Sixty teachers from over 40 institutions of higher learning participated in the 2-day conference.

After the conference, several institutions asked permission to use the "Companion" in courses during the 1992-1993 academic year. Those who are using it and providing class-testing information (in addition to Moravian College and Northampton Community College) are: Amherst College, Bates College, Lehigh University, Mount Holyoke College, Ohio State University, Randolph Macon College, and Saint Olaf College.

In September 1993, a publishing contract was signed with Brooks/Cole. The text is currently in production (in "preliminary" form so that another year's class-testing can take place) and will be available for adoption for Fall 1994 classes. Dissemination activities will continue even though the FIPSE support has ended.

**Evaluation/Project Results**

The purpose of our project was to provide materials for a slower-paced introductory calculus course (Calculus I with Review) that aimed to overcome the inadequate preparation of our students. The goals were to increase student understanding, improve morale, and attain a higher rate of completion of the Calculus I course by these students. To assess how well we accomplished these goals in the two years of the project, we carried out a four-pronged evaluation, each measuring a different aspect. The full evaluation report with detailed data is in Appendix 2.

(1) Data on course enrollment, attrition, and completion of the Precaclulus-Calculus I sequence for the years 1986–1988 (2 years prior to the integrated course at Moravian) and similar data for the Calculus With Review (Math 106) course 1988–1993 shows an improvement in the percentage of underprepared students who complete Calculus I.

(2) A comparison of student performance on 13 precalculus questions before taking the course and after taking the course shows substantial improvement in understanding.

(3) A comparison of performance by students in the "integrated" course with those in the "regular" course on 25 calculus questions showed that students in the "regular" course outscored those in the integrated course on almost all questions, but the differences in many cases were only slight. The adult students in the evening section of the integrated course did considerably better than "traditional" students in other sections of the integrated course, and on several questions, outperformed students in the "regular" course.

(4) A survey was designed to evaluate student attitudes toward learning mathematics, perception of what was important in the course, opinions about the "Companion" materials, the calculus text, and the course. In general, the responses show that there was a perceived improvement in ability to learn mathematics, that the "Companion" was helpful (and read more often than the text), and that mathematics was seen as important in other areas. In addition, the topics that were chosen most frequently as being important ones in the course were those that the faculty members also deemed fundamental.

**Summary and Conclusions**

The immediate goals of the project have been realized: the supplemental text, "A Companion to Calculus" has been produced, revised, and class tested, and is currently in use in 9 different institutions. A preliminary edition will be commercially published in the next year. We have made efforts through presentations, conferences, and articles to disseminate information about our project and interest others in the concept of an integrated course. These efforts will continue.

We have gained several insights as a result of this FIPSE-funded project. Although we have met the immediate goals of the project, it has taken more time and been more difficult than we envisioned. The desk-top publishing of the text for class-testing also was far more difficult than envisioned, primarily because of inadequate technical support, but also because of time constraints. Close cooperation among the four faculty in the "FIPSE group" was essential to the success of the project. In addition, in order to accomplish the project goals, it was necessary to have the cooperation and support of other colleagues—in teaching the course, class-testing materials, and participating in evaluation efforts.

We feel, and our evaluation report supports this feeling, that our project has improved access to calculus for poorly prepared students. However,we continue to recognize that the problem is one that cannot be solved with a single course, regardless of the materials and teaching efforts.

Body of Report

## Project Overview

Our project began as a response to a serious problem for those who teach calculus: roughly half of the students required to take calculus are inadequately prepared. The standard response of most institutions is to offer a one-semester course called "Precalculus" (or "College Algebra and Trigonometry") and require such underprepared students to successfully complete this course before they are admitted to a calculus course. Until 1988, Moravian College followed this standard pattern and taught a Precalculus course. Attrition in the course was high, morale (of both students and teachers) was low, and the purpose of the course was not realized: students did not remember the preparatory material when needed the next semester in calculus.

In 1988, in an attempt to address these problems, a new one-year course "Calculus I With Review" was introduced at Moravian that replaced the Precalculus-Calculus I sequence for those students not ready to begin Calculus I. The course integrates the review of precalculus topics *as they are needed* within the first calculus course. Although many Precalculus books are available, there are no published materials that are designed to be used in such an integrated course. While designing our course, we found that colleagues in other institutions had a strong interest in our integrated approach, but the impediment to their offering such a course was a lack of supplemental materials for the review portion of the course. Our project was to prepare, desk-top publish, and class-test supplemental materials especially designed to integrate the review of algebra skills, functions, graphing, and problem-solving techniques in the beginning calculus course.

Our 2-year funding by FIPSE began on August 20, 1991. Throughout the academic year 1991-1992, three faculty members from the mathematics department at Moravian College (Kay Somers, Alicia Sevilla, and Doris Schattschneider) and a faculty member from the mathematics department of Northampton Community College (Dennis Ebersole, also an adjunct at Moravian College who had taught the integrated course) developed and desktop-published the first draft of the supplemental course materials. Entitled "A Companion to Calculus," they were used as an integral part of the course in all sections of Calculus I with Review at Moravian College. During the summer 1992, a revised draft of the "Companion" was prepared that incorporated corrections, additions and suggestions that resulted from class-testing and evaluation. During the 1992-1993 year, this revised draft was used at Moravian College for all sections of Calculus I with Review as well as in a one-semester Calculus I course at Northampton Community College, where it served as a supplement for students to use on their own. A copy of the revised draft accompanies this report as Appendix I.

Evaluation of several aspects of the project was ongoing throughout the 2-year project period: records on student enrollment and completion of the course as well as the former sequence

it replaced; comparison of individual student performance on pretests and post-tests; comparison of Calculus I with Review student performance on selected exam questions with those in the "regular" one-semester course; attitude surveys of those in the integrated course; evaluation of the materials by students and instructors. The full evaluation report is in Appendix 2.

Dissemination was also a major part of the project. Although at the outset of the project we identified a few other institutions who had a first calculus course that integrated precalculus review, these were the exception to the rule. We wanted others who were dissatisfied with the traditional Precalculus-Calculus sequence to know about our integrated approach and our "Companion." In June 1992, Kay Somers and Alicia Sevilla reported on the project at the Conference on the Teaching of Calculus held at Harvard University. Their report was solicited and was published in PRIMUS, Volume III, no. 1, March 1993, pp. 35–41. In January 1993, at the national MAA-AMS meetings in San Antonio, Doris Schattschneider and Dennis Ebersole participated in a 4-hour poster session devoted to Calculus reform projects. In April, 1993, Doris Schattschneider was an invited participant and panelist at an NSF-sponsored conference on Precalculus and Calculus Reform held at the University of Illinois Allerton Conference Center in Monticello, Illinois. Her description and report on the FIPSE project will be in the Proceedings of the conference, published by the Mathematical Association of America. On June 18–19, 1993, a dissemination conference was held at Moravian College, organized by the project participants. Sixty teachers from over 40 institutions of higher learning participated. About 15 of the institutions represented have "integrated" courses in place; many others plan to inaugurate such courses. The announcements and program of the conference are in Appendix 3.

As a result of the dissemination efforts, the second draft version of the "Companion" is being used at 9 colleges and universities during the 1993-94 academic year. These are a mix of small colleges and large universities, and we look forward to their comments and feedback. Several other inquiries have come from those interested in using the materials and/or developing an integrated course at their institutions. The dissemination efforts also brought keen interest in the "Companion" from several publishers. One of the key features of the "Companion" is that it is designed to be used with any standard calculus text, and is adaptable for use in a variety of calculus courses. In September, 1993, we were offered contracts by two different publishers, and finally signed with Brooks/Cole. A published "preliminary" edition of the "Companion" will be out in early 1994, ready for adoption for courses in Fall 1994. The publisher is enthusiastic about disseminating information about the use of the text in an integrated course. Even though FIPSE funding has ended, we will continue our dissemination efforts.

## Purpose

The purpose of the project has been described both in the overview above and in detail in the section on Background and Origins below. In short, our goal was to produce materials to use in an integrated one-year course that integrates the review of needed precalculus concepts and skills with the introduction of calculus. The larger purpose was to provide access to calculus to those who are underprepared, and to try to address the problem of poor skills, attitude, and morale that is pervasive in a course required of students who have never done well in mathematics. In addition, we aimed at making our approach known to those at other institutions of higher learning who were dissatisfied with the traditional route of remedial courses in Precalculus (or similar courses) prior to a briskly paced one-semester first course in calculus.

In writing the materials, we made a concerted effort to try to identify the main weaknesses in mathematical skills and understanding that prevent students from succeeding in calculus. The materials were to specifically address these needs, and do this in the context of learning calculus; they were not to be just another survey of precalculus topics. We also made a deliberate effort to present explanations in four modes: words, symbols and formulas, pictures and graphs, and numerical data, and to require students to answer questions in these different modes.

Although our evaluation indicates that we were successful in producing materials that most students found readable and helpful, the materials alone cannot address the problem of turning around long-standing attitudes toward mathematics and study habits that lead to failure. Those who taught the course tried different teaching strategies in class, as well as provided extra tutoring help outside of class. For those students who made the effort (and this includes all of the highly motivated adult students), great progress was made. Both students and teachers were encouraged by the improvement in mathematical performance and growth in self confidence of these students. But many could not be turned around, at least in the freshman year. This is a much larger problem that needs to be addressed long before students arrive in college, as well as by those teaching in college. We still have much to learn about how to try to reverse attitudes and actions that contribute to failure in mathematics.

## Background and Origins

In recent years, as mathematical techniques have gained increasing importance in a wide variety of fields, the requirement of calculus has become standard not only for the physical and mathematical sciences, but for many other college majors. This has created a serious problem for those who teach calculus. At Moravian College, 60% of the freshman class intend to pursue a course of study that requires calculus. Our problem is that roughly half of these students are inadequately prepared to take calculus. Their high school record in mathematics and a college-administered placement examination show that they cannot possibly succeed in a calculus course

without improving their understanding of basic algebra concepts and their problem-solving skills. This problem is widespread; there are few (if any) institutions of higher learning in the United States that do not share it. A 1985-86 survey on Undergraduate Programs in the Mathematical Sciences published by the Conference Board of the Mathematical Sciences reported that remediation in mathematics was ranked as the most serious problem in two-year mathematical sciences programs. It was rated as a major problem by 39% of universities, by 66% of four-year public colleges, and by 45% of four-year private colleges. For these institutions, remediation for calculus is a significant problem. In the years since the survey, the situation has not improved.

A few highly selective schools choose to address the problem indirectly by advising students to take a remedial course elsewhere before enrolling in calculus; some schedule non-required workshops or provide tutorial help for students in the calculus course. However, the standard response of most institutions is to offer a one-semester course called "Precalculus" (or "College Algebra and Trigonometry") and require such underprepared students to successfully complete this course before they are admitted to a calculus course. Until 1988, Moravian College followed this standard pattern and taught a Precalculus course. Each Fall, to serve the number of students who would need calculus for their intended major, there were typically 4 or 5 sections of Precalculus with 25-30 students each (including an evening section in the Division of Continuing Studies) and the same number of Calculus I sections for those students ready for calculus. (In the spring term and in the summer session there was also at least one section of each of the courses.) Those students who were not adequately prepared to take Calculus I were primarily those who intended to major in business and economics or biology, but there were also some who planned majors in other social sciences or the mathematical, physical or computing sciences.

The morale of both students and teachers in the Precalculus course was low; the attrition in the course was fairly high; the retention of material needed for Calculus the next semester seemed extremely low. (The attrition figures can be found in our Evaluation Report, Appendix 2.) These observations are typical— almost any conversation with colleagues at other institutions about the problem rehearses the same litany of woes. There are many fairly obvious reasons for these occurences. Some of the morale problem no doubt stemmed from the fact that students were in a required course that they would not otherwise choose. But a large part of it is because it is disheartening and boring for both students and instructors to spend a whole semester redoing the necessary "precalculus" topics already seen (but not learned) in high school with only a promise that students will "need these ideas and skills next semester" when they take calculus. There is little, if any connection to the calculus— students still have no idea of what this "calculus thing" is, and rarely learn why these precaclulus topics are necessary. Their view is that mathematics is a large set of unconnected rules to memorize and that to master this is an impossible task. Perhaps it

is not too surprising that when, in Calculus I, these students are expected to know how to apply these concepts and skills in the context of solving a calculus problem, they don't know what to do.

The marked attrition rate within Moravian's Precalculus course (those that withdrew or failed) and the high drop-off of those who completed Precalculus but did not enroll in Calculus I is also a typical occurrence at most other institutions. In fact, Moravian's attrition rate was somewhat low compared to that in larger institutions; there the percentage of students who begin with Precalculus and complete Calculus I in the same year may be 10% or lower. It is an important sign that the Precalculus course may not be the best way to help students succeed in calculus. In fact, it would appear that it is a significant factor in ensuring that many students never make it to the calculus course.

In 1987, the Mathematics Department at Moravian proposed to the faculty that the Precalculus course be dropped and a new one-year course, Calculus I with Review, replace the Precalculus-Calculus I sequence for those students who were not prepared to begin with Calculus I. The time investment for students in the course would be the same, but the way in which background material was reviewed would be markedly different. It might be noted here that Moravian College is a liberal arts College with approximately 1200 undergraduates and 85 full-time faculty members, with 6 in the Mathematics Department. The size and collegiality of the department made it relatively easy for the whole department to support this proposed new course; indeed, the whole department worked out the formulation of the syllabus. Convincing colleagues from other departments that this change would be for the better and not cause major problems was not too difficult. Although there was spirited discussion about the proposal, the faculty endorsed the change, and in 1988 the new course was taught for the first time.

The course integrates the review of precalculus topics as they are needed within the first calculus course. The same calculus text and calculus syllabus are used in the one-semester Calculus I course and in the slower-paced one-year course. But in the one-year course, concepts and skills in algebra, functions, graphing, and problem-solving are reviewed in conjunction with the treatment of new concepts and techniques of calculus. In the first two years the integrated course Calculus I With Review was offered, there was an observed improvement in morale, level of understanding, and the number of students completing the course.

The greatest difficulty in teaching the integrated course was the lack of published material to provide the needed background and review, but in a calculus setting. Although many Precalculus books are available, there are none that are designed to be used as part of a calculus course. We used some published "refresher" college algebra and precalculus texts, but found that they did not contain the specific topics needed to integrate well with the calculus topics, and so decided that if the course was to continue (and we wanted it to), we would have to prepare our own supplemental materials. It was then (1990) that we wrote the grant proposal to FIPSE to support the preparation

and production of the supplemental materials, a careful evaluation of the course and effectiveness of the materials, and dissemination activities for the project. We requested support for released time from teaching during the academic year, for summer work, for typing and printing the materials, and for implementation of evaluation and dissemination activities. Dennis Ebersole, then chair of the mathematics department at Northampton College had taught one section of the integrated course at Moravian in the Division of Continuing Studies. He indicated strong interest in joining the project, both as a writer and teacher, and hoped that the course could be emulated at Northampton Community College.

Although a great deal of time, effort, and funding by Federal agencies was focused at that time on various calculus reform projects, the primary emphasis of these other projects was to change the way calculus was taught. The subject of access to calculus, and the need to address how underprepared students can best receive the necessary remediation to succeed in calculus was not directly addressed by these reform projects.

In writing our proposal, we sought and received promises of support from Moravian College and Northampton Community College for facilities and resources for our work and for the planned dissemination conference, as well as permission for released time from teaching during the project period. In addition,the Lehigh Valley Association of Independent Colleges pledged support for the planned dissemination conference. We also sought and received letters of interest in the proposed materials from two publishers (Addison Wesley and Wadsworth-Brooks/Cole). Interest from publishers increased during the grant period, and we received support from two publishers (Addison Wesley and W.H. Freeman) for receptions at our dissemination conference.

## Project Description

The main features of the project have been described above in the Project Overview. In this section we give details of several aspects of the project during the 2-year project period.

Our funding began on August 20, 1991. During the last two weeks of August 1991, project team members met for several hours daily and outlined the supplemental material to be written for the year-long course. In addition, materials for roughly the first four weeks of the course were prepared and duplicated. In this initial phase, we established a working pattern that was followed throughout the year. For each topic in the calculus course, the group of four would discuss the pitfalls that were typical for students with weak background, identify possible ways to address these items, then assign the writing of the first draft of a chapter to one of three team members. (It was agreed that Doris Schattschneider, the project director, would serve primarily as editor and administrator, and not write first drafts.) These first drafts were then heavily critiqued in open discussion by the team and revised drafts prepared.

At the outset, the team agreed that the information in the materials would be presented through four different modes: words (descriptive sentences), figures (graphs and diagrams), symbols (algebraic formulations), and numerical data. We agreed that students responded differently, with varying degrees of understanding, to each of these different ways of presentation, and that in order to achieve the greatest understanding, it was important to use all modes. We also wanted students to be able to "translate" information from one mode to another. We agreed that the text would teach primarily through examples, and contain numerous exercises, the majority of which encourage or require visual presentation and interpretation of information. We wanted the materials to encourage the use of technology, particularly graphing calculators or graphing programs for computers, but not depend on this technology. We also agreed that since the materials were to be used with a calculus text, standard terminology and notation from calculus would be used while providing background, rationale, and development of skills necessary for understanding and solving calculus problems.

During the Fall semester, two team members (Dennis Ebersole and Alicia Sevilla) had released time of one course each to continue to write materials for the course. The team (Ebersole, Sevilla, Kay Somers, and Schattschneider) met often (every week to 10 days) to critique drafts of completed chapters and to discuss ideas for new chapters. Throughout the term, the materials, entitled "A Companion to Calculus" were distributed to students in all sections of Moravian's Calculus I with Review and used as an integral part of the course. The calculus text for the course (as well as for the "regular" one-term course) was *Calculus, Early Trancendentals version* , by James Stewart, Brooks/Cole, 1991. Those teaching a section of the course were Sevilla, Somers, Ebersole, and Mohamed Bugaighis. Seven chapters of the "Companion" were completed and used in classes during the Fall term. During the Fall term, the team also formulated its evaluation plans for the project, and in January 1992, the first evaluation data was gathered and analysed (see section below for details).

In January, 1992, when three of the four project participants were not teaching (Moravian College had a separate January term), the team met very frequently to prepare additional materials and to reformulate the outline of materials through the end of the course. During spring term 1992, Kay Somers had released time from one course to write materials and Dennis Ebersole and Alicia Sevilla were back to full-time teaching. That spring, those teaching a section of the course were Sevilla, Schattschneider, Ebersole, and Bugaighis. The team continued to meet as in the Fall, to discuss, critique, and review drafts, and the materials were produced and class-tested on a very tight schedule. Constraints of time and money made it necessary to have project writers produce the material on their own computers, and utilize the wordprocessing and graphing software available to them. The materials were duplicated at a local printing company and produced on three-hole punched 8 1/2 x 11 inch paper so that they could be easily added to the

looseleaf binder volume whose size grew to over 600 pages. At the end of the Spring term, an evaluation similar to that done at the end of the Fall term was carried out and an evaluation report for the first year was prepared by Schattschneider.

The first draft of the "Companion" produced during the 1991-1992 year lacked uniformity of style and appearance: for the most part, the materials had been draft-typed on a variety of wordprocessors by the writers themselves. Graphs and diagrams (an important feature of the materials) had also been produced in a variety of ways, and were not consistent in appearance. Throughout the year, all those who taught the course using the "Companion" materials noted corrections, suggestions (what didn't work), additions (what got left out), and in general made notes to be used in preparing a revised version of the materials. The summer of 1992 was spent in making major revisions of the "Companion" and producing a volume with fairly uniform appearance and style. The revised text, containing 20 chapters, plus solutions to exercises, is more than 700 pages. The table of contents follows; Appendix I of this report is the complete text.


Chapter 0. Introduction: Symbols and Notation, Modes of Communication.

Chapter 1. Cartesian Coordinates: A Cartesian Coordinate Plane, Graphs, Lines and Their Equations, Parallel and Intersecting Lines, Distance Between Two Points, The Circle.

Chapter 2. Functions: Function Notation, Domain and Range of a Function, Different Ways to Represent Functions, The Graph of a Function, Special Classes of Functions, Transformations of Graphs.

Chapter 3. Companion to Limits: Algebraic Combinations of Functions, Algebraic Simplification of Functions (Quotients of Polynomials, Quotients with Radicals, Complex, Quotients with Absolute Values), Inequalities (Linear Inequalities, Absolute Value: Equations and Inequalities), If-Then Statements.

Chapter 4. Companion to Continuous Functions: Polynomials, Zeros of a Polynomial Function (Finding Zeros Exactly, Linear Factors of Polynomial, Approximating Zeros of a Polynomial Function), Composition of Functions, Domains of Functions).

Chapter 5. The Role of Infinity: Graphical Interpretation (Horizontal Asymptotes,Vertical Asymptotes), Algebraic Manipulations (Finding Horizontal Asymptotes, Finding Vertical Asymptotes).

Chapter 6. Rates of Change: Problem Solving, Applications (Rates; Average Rate of Change), Secant and Tangent Lines.

Chapter 7. Companion to Rules of Differentiation: Negative and Rational Exponents, Decomposition of Functions

Chapter 8. The Trigonometric Functions: Angle Measures, Definition and Evaluation of Trigonometric Functions, Exact Values of the Trigonometric Functions for some Special

Angles, Properties of the Trigonometric Functions, Domain, range, and graphs of the Trigonometric Functions, Combining Functions with the Trigonometric Functions.

Chapter 9. Companion to Implicit Differentiation: Implicitly Defined Functions, Solving Equations containing $dy/dx$.

Chapter 10. Companion to Repeated Differentiation: Iteration, Rate of Change of Rate of Change.

Chapter 11. Companion to Related Rates: Setting up Equations for Related Rates Problems, Problem-solving Strategies for Related Rates Problems.

Chapter 12. Linear Aproximations and Differentials: Tangent Line Approximation, The Differential.

Chapter 13. Companion to Exponential Functions: Rules of Exponents, The Natural Exponential Funtion

Chapter 14. Companion to Inverse Functions: One-to-one Functions, Inverse of a Function (Domain and Range of $f$ and $f^{-1}$, Graphs of Functions and Their Inverses), Finding the Inverse.

Chapter 15. Companion to Logarithmic Functions: Definition and Properties of Logarithmic Functions, Graphs of Logarithmic Functions, Solving Equations with Logarithmic and Exponential Functions.

Chapter 16. Companion to Extreme Values of a Function: Solving Equations to Find Critical Values, Setting up Functions to Solve Extreme Value Problems.

Chapter 17. Companion to Curve Sketching: Solving Inequalities, Graphical Interpretation, Putting it All Together.

Chapter 18. Companion to Antidifferentiation: Antidifferentiation as the inverse of differentiation, Finding Antiderivatives, Substitution for Antiderivatives.

Chapter 19. Companion to Area Under a Curve: Computing Exact Areas Using Basic Geometric Shapes, Approximation of Areas, The Area Under a Curve as a Definite Integral.

Chapter 20. Companion to the Definite Integral: Interpretations of the Definite Integral, Companion to the Fundamental Theorem of Calculus, Change of Variable in Definite Integrals.

Appendix: Solutions to exercises.

With the exception of a couple of chapters, all chapters were typed by Kathy Burkert, the mathematics department secretary at Moravian. Some graphs were prepared by a student using a computer graphing program (Quatro Pro), directed by the authors of the chapters. It should be noted that the "desktop" production of the materials was far more difficult than we had anticipated, and took far more effort and time than planned. The lack of a true scientific wordprocessor made

the typing of formulas and equations cumbersome and not uniform (WordPerfect, with its equations editor, was used for most chapters), and the lack of interface with a good graphing program for functions made the production of graphs a real headache. Most graphs were done many times to achieve desired results; often the wordprocessor would not "paste" them into the text as desired, and sometimes hand touch-up and scissors and paste corrections were the only solution to problems.

During the academic year 1992-1993, the revised draft of the "Companion" was used in all sections of Moravian's Calculus I with Review, along with Stewart's *Calculus*. In addition, Dennis Ebersole used it as a required supplement to a one-semester Calculus I course at Northampton Community College. We welcomed this alternate use of the "Companion" as a test of its flexibility to be used in a different situation and with a different calculus text (*Calculus*, by Ross Finney and George Thomas, Addison Wesley, 1990). Although Ebersole did not carry out a formal evaluation of his course, he reported that the students liked the supplement and some students from other sections not using the supplement requested to buy the supplement. It should be noted that Ebersole had originally hoped that Northampton Community College would adopt the Moravian model and replace its Precalculus-followed-by-Calculus I sequence with an integrated course, but there was strong opposition to making that change, at least at that time. (Other changes to the Community College mathematics curriculum were being discussed and implemented.)

The writing team met only as necessary during the 1992-1993 year. During the Fall term, it met to discuss and review drafts of three short chapters that had been planned, but not produced at the end of the previous Spring term. It also met to discuss the second year evaluation plans. (Details on evaluation are contained in the next section of this report.)

In addition to class-testing the revised materials, a primary focus of the project during the 1992-1993 year was dissemination activities. We felt that dissemination activities were premature until we had at least taught the year-long integrated course with the "Companion," done an evaluation, and prepared a revised draft of the "Companion." In our original proposal to FIPSE, we had anticipated a dissemination conference to be held in the summer of 1992. As the project progressed during its first year, it was apparent that we would not be adequately prepared for such a conference until the summer of 1993. We sought and received permission from FIPSE and LVAIC (who had agreed to support the conference with $1000) to postpone the conference.

The purpose of dissemination was several-fold: to make known our integrated course and "Companion" text to support it, to provide information about our course and materials to those interested in reform, to identify other institutions currently teaching such a course or interested in offering such a course in order to see if they would like to class-test the "Companion," to initiate discussion concerning alternative approaches to making calculus accessible to students who are not

prepared for it, and to have the "Companion" commercially published and promoted to make it readily available to all colleges and universities.

Information about our project was first disseminated through the project director's description prepared for the FIPSE Project Directors' meeting in Fall 1991, and a panel at that meeting on mathematics and science projects in which Schattschneider described the project. Several institutions in attendance contacted us for further information. The following June, Kay Somers and Alicia Sevilla made a presentation about our project at the Conference on the Teaching of Calculus held at Harvard University. Their report described the project and illustrated with several examples the flavor of the "Companion." Their report was published in a special issue of the journal PRIMUS that was devoted to Calculus reform and the Harvard Conference (Volume III, no. 1, March 1993, pp. 35–41; see Appendix 4.).

During the second year of the project, more than 40 copies of the "Companion" were sent to those who requested it. These requests were the result of contacts made the previous year, and in large part, the further dissemination activities in 1992–1993. At the fall 1992 FIPSE Project Directors' meeting in Washington, D.C., Kay Somers made contact with others interested in the project. In January 1993, at the national MAA-AMS meetings in San Antonio, Doris Schattschneider and Dennis Ebersole participated in a 4-hour poster session devoted to Calculus reform projects. At that session, approximately 40 different calculus reform projects were showcased; our FIPSE project was the only one concerned primarily with access for the underprepared student. Many people discussed the project with us there, and many requested further information on the project and our announced dissemination conference. We also initiated conversations with publishers. During Spring, 1993, there were seven different publishers who showed interest in the "Companion" and had it reviewed: HarperCollins, Saunders, Brooks/Cole, Addison-Wesley, Prindle Weber Schmidt, W. H. Freeman, and West.

In April, 1993, Doris Schattschneider was an invited participant and panelist at an NSF-sponsored conference on Precalculus and Calculus Reform held at the University of Illinois Allerton Conference Center in Monticello, Illinois. She was one of four participants in the panel on precalculus reform, and the only one who indicated that, at the college level, reform could mean the elimination of Precalculus as a separate course. Her description and report on the FIPSE project will be in the Proceedings of the conference, published by the Mathematical Association of America; see Appendix 5.

During the fall and winter of 1992-1993, the team made plans for the dissemination conference "Integration of Precalculus with Calculus" to be held at Moravian College June 18–19, 1993. Dr. Susan Forman, of the Mathematical Sciences Education Board, agreed to present an overview of reform projects and raise the many questions that must be faced by individuals and institutions undertaking reform. Four individuals from institutions with integrated courses agreed

to be on a panel to discuss their integrated courses and answer questions. The panelists were Prof. Lenore Frank (SUNY, Stoneybrook), Prof. Nancy Baxter (Dickinson College), Prof. Michael Rogers (Amherst College), and Prof. Dennis Ebersole (Northampton Community College and also representing Moravian College). Also, contributed papers were solicited.

Announcements for the conference were published in several regional and national mathematical newsletters; in particular, FOCUS (the MAA newsletter) and the NOTICES (the American Mathematical Society news journal), and the AWM (Association for Women in Mathematics) newsletter carried the announcement. This generated not only interest in the conference, but many inquiries about the project. In June, 60 teachers from more than 40 institutions of higher learning participated in the 2-day conference. About 15 of the institutions represented had "integrated" courses in place; many others hoped to inaugurate such courses and were there to learn. There were 12 contributed papers. Two publishers were represented at the conference (Addison Wesley and W. H. Freeman & Co.). In addition to the scheduled program of the conference, there was continuing informal discussion among the participants about the common concern of offering courses that provide access to calculus for those with inadequate preparation. Moravian's "Companion" and evaluation report of the FIPSE project were made available to participants at the conference. See Appendix 3 for announcements and the program of the conference.

After the conference, we received several requests from other institutions to use the "Companion" in courses during the 1992-1993 academic year. Those who are using it (in addition to Moravian College and Northampton Community College) are: Amherst College, Bates College, Lehigh University, Mount Holyoke College, Ohio State University, Randolph Macon College, and Saint Olaf College. Information is being gathered from all these institutions on the number and types of students served, the type of courses, the way in which the "Companion" is being used, and also general reactions to the "Companion" by students and instructors. In addition we have requested (and are receiving) comments, corrections, and suggestions on the "Companion."

In August, 1993, two publishers, Addison Wesley and Brooks/Cole each (separately) had extensive discussions with us about the publication of the "Companion" and plans to further widespread dissemination of its use. In September, we decided that Brooks/Cole was the best choice for our text and signed a publication contract. The text is currently in production (in "preliminary" form so that another year's class-testing can take place) and will be available for adoption for Fall 1994 classes. The publisher is enthusiastic about disseminating information about the use of the text in an integrated course.

Dissemination activities will continue even though the FIPSE support has ended. Alicia Sevilla and Kay Somers will make a presentation at a session on "The Bridge to Calculus" at the national MAA meeting in Cincinnati in January 1994. Dennis Ebersole has been invited to make a

presentation about the project at the joint 1994 regional Spring meeting of the MAA and AMATYC (Association of Mathematicians at Two Year Colleges) in Harrisburg. We will continue to respond to inquiries and interest in the project, and will work with the publisher to disseminate information about our text and project.

**Evaluation/Project Results**

The primary purpose of our project was to provide materials for a slower-paced introductory calculus course (Calculus I with Review) that aimed to overcome the inadequate preparation of our students. The goals were to increase student understanding, improve morale, and attain a higher rate of completion of the Calculus I course by these students. To assess how well we accomplished these goals in the two years of the project, we carried out a four-pronged evaluation, each measuring a different aspect. Our evaluation sought to measure both quantitative and qualitative data. The four parts of the evaluation are described below. The full evaluation report with detailed data and displays is in Appendix 2.

(1) Data was gathered on course enrollment, attrition, and completion of the Precalculus-Calculus I sequence for the years 1986–1988 (2 years prior to the introduction of the integrated course at Moravian) and similar data for the Calculus I With Review (Math 106–166) course 1988–1993. This data shows that in the last four years, while the percentage of students who enroll in the Calculus I with Review and complete the first semester is approximately the same as that for the Precalculus course offered before 1988, there is a significantly larger percentage of students who now continue the second semester and complete Calculus I. The figures that were recorded for this part of the evaluation only counted students who completed the integrated course in a single year. There are several students each year who drop out midyear, or even during the second semester, and then complete the course in the Spring of the following year.

(2) Every entering freshman takes a "high school mathematics assessment" (HSA) as part of the orientation and preregistration process at Moravian. This 45-minute examination of precalculus skills was devised several years ago by the Mathematics Department. It is one measure that is used to help place students who need calculus for their intended course of study. In order to see if students improved their understanding of precalculus material, a comparative evaluation was made. At the end of the Fall term in both 1991 and 1992, thirteen questions from the HSA were also on the final examination for Math 106, the first part of Calculus I with Review. Grades for each freshman on each question on each of the two tests were recorded and compared. The table of these comparative grades shows that in every case, there was significant improvement in the number of students who gave correct answers. The table records, for each of the 13 questions, the number of students in each of the 7 categories: correct on the HSA; correct on Final; correct on both HSA and Final; incorrect on both HSA and Final; correct on HSA and incorrect on Final

(grade down); same on HSA and Final (same grade); incorrect on HSA and correct on Final (grade up). The table also makes clear on which of the 13 questions students improved least and improved most. (Following the table, the report reproduces the questions.)

(3) In order to see how the understanding of students in the integrated Calculus I with Review course compared with those in the "regular" one-semester course, 25 questions that tested calculus material common to both courses were on the final examinations of both courses. This was done in each of the 2 years of the project. For the Calculus I with Review students, 12 of these questions were on the examination the end of the Fall term, and 13 questions were on the examination at the end of the Spring term. The mean and median scores of each group were computed for each question for purposes of comparison, as well as standard deviations for these scores. Not surprisingly, students in the "regular" course outscored those in the integrated course on almost all questions, but the differences in many cases were only slight. The adult students in the evening section of the integrated course did considerably better than "traditional" students in other sections of the integrated course, and on several questions, outperformed students in the "regular" course.

(4) Perhaps the most persistent problem in teaching students in the integrated course is their attitude and self-image with regard to learning mathematics. In order to access whether change had taken place as a result of the course, an evaluation instrument was developed by the team with the assistance of Dean William Deeds, a social scientist experienced in the area of testing and evaluation. The survey was designed to evaluate student attitudes toward learning mathematics, perception of what was important in the course, opinions about the "Companion" materials, the calculus text, and the course. For the 2-year period, this survey was administered at the end of each of the terms to all students in Calculus I with Review. The summary of responses to each question is presented as a histogram in the evaluation report. In general, the responses show that there was a perceived improvement in ability to learn mathematics, that the "Companion" was helpful (and read more often than the text), and that mathematics was seen as important in other areas. In addition, the topics that were chosen most frequently as being important ones in the course were those that the faculty members also deemed fundamental. While the results of the survey are generally encouraging, it should be noted that there were several negative comments; some students were unswayed in their dislike of mathematics and feeling that it was a useless imposed burden.

The evaluation required cooperation of all members of the mathematics department. The greatest cooperation was needed for planning the common questions for the final examinations of all sections of the two different courses. All members of the department met to discuss and decide on the specifics of this common element of the examinations, as well as agree on a common way to grade the questions. (Each faculty member grades his or her own final examination.) In addition,

all faculty teaching the integrated course were asked to administer the student opinion survey at the end of each semester.

The compilation and analysis of all the data for the evaluation was a time-consuming task that was carried out in the first year by Dawn Voorhees and in the second year by Matthew McShea. Each of these students was a mathematics major familiar with statistical methods, and the data analysis was prepared using the program *Minitab*. The final evaluation report was prepared by Doris Schattschneider.

Although Moravian College will continue to monitor enrollment and completion data for the course, we do not plan to continue the detailed comparative evaluation of students' performance on test questions that was part of our project evaluation.

**Summary and Conclusions**

The immediate goals of the project have been realized. The supplemental text, "A Companion to Calculus" especially designed for a slower-paced one year Calculus I course that integrates the review of precalculus topics as they are needed has been produced, revised, and class tested, and is currently in use in 9 different institutions. A preliminary edition will be commercially published by Brooks/Cole in the next year, and in the following year, a "first edition" will be produced. We have made efforts through presentations, conferences, articles and our own dissemination conference to disseminate information about our project and to interest others in the concept of an integrated course. These efforts will continue.

We have gained several insights as a result of this FIPSE-funded project. Although we have met the immediate goals of the project, it has taken more time and been more difficult than we envisioned. We would advise anyone undertaking such a project to seek support for at least three years, including two summers. While our funding ended in August 1993, we will be very busy this 1993-94 year as the "preliminary" version is commercially produced. At the same time, we will solicit and gather information from the other institutions who are using the text so as to prepare a revised version for publication in 1995.

The desk-top publishing of the text for class-testing also was far more difficult than envisioned, primarily because of inadequate technical support, but also because of time constraints. The mode of writing that we adopted (discuss, write, tear apart, rewrite, edit) was a very helpful one, but was time-consuming. In addition, it tested the friendships and tolerance of the "FIPSE group" of four—all had to be willing to entertain each other's ideas and criticism. The group worked well with each other and are still friends. However, any group undertaking this sort of project must begin with participants who respect each other and are willing to give and receive criticism.

Cooperation is also required from the department members who teach the course, but are not part of the project team. We were very fortunate to have a collegial group of faculty members who not only supported the new course and were willing to class-test the materials, but were also willing to cooperate with the evaluation plan. Without this cooperation, a project such as this would be impossible.

We feel, and our evaluation report supports this, that our project has improved access to calculus for poorly prepared students. At the outset, we were optimistic that improvement in the numbers of these students completing Calculus I would be dramatic, and that the negative attitude of most of these students towards mathematics would be turned around. Our gains in these areas were more modest than hoped, and we continue to recognize that the problem is one that cannot be solved with a single course, regardless of the materials and teaching efforts. We are very encouraged by the positive response of adult students to the materials and the course.

Appendix. Comments to FIPSE

The project directors' meetings in the fall of each year were very helpful. At these meetings, good contacts were made with others interested in similar problems or who had complementary projects. Also conferences with FIPSE personnel to answer questions about project administration and evaluation were helpful. These conferences were well-planned.

Telephone access to our program officer, Brian Lekander, was good, and he was helpful in providing advice when needed. Brian indicated during both years of our project that he intended to make a site visit, but this was never done. We were disappointed that no one from FIPSE attended the dissemination conference.

One frustration of working with FIPSE was the delay in anticipated start date of our funding, and as a result, the necessity to revise our schedule for the project. This in turn made necesssary a formal request for a supplement to our first year budget, taken from our second year budget. It would be helpful if FIPSE's formal approval process could take into account the nature of the project and the importance of certain start dates for planned projects. It also proved frustrating not to have the flexibility to easily transfer budgeted funds from one category to another when anticipated costs differed by more than a trivial amount. On the basis of having worked with other Federal funding agencies, it would seem that FIPSE allows the project director far less flexibility in administration of project funds within the approved budget. It would be helpful if the director were given more flexibility in this area for small changes, and if the process of making larger budget changes from one category to another (not increasing the budget) could be handled by a simple request, giving justification.

The problem of access to mathematics (and calculus in particular) at the college level is one that continues. The number of remedial courses taught at colleges and universities (especially public 2-year and four-year institutions) and numbers of students enrolled in these courses is a scandal. FIPSE should encourage projects that try to address this problem in new ways at any level (pre-college and college).

Appendix 1: 1992 draft of text *A Companion to Calculus*

Appendix 2: Evaluation Report of the two-year project

Appendix 3: Announcements and program of the dissemination conference "Integration of Precalculus with Calculus" held at Moravian College, June 18-19, 1993

Appendix 4: "Integrating Precalculus Review with the First Course in Calculus," Alicia Sevilla and Kay Somers, PRIMUS, vol. III, no. 1, 1993, pp. 35–41.

Appendix 5: "A Report on a Project to Develop Course Materials to Integrate Precalculus Review with the First Course in Calculus," Alicia Sevilla, Kay Somers, and Doris Schattschneider, Proceedings of the Conference on Calculus and Precalculus Reform (Monticello, Illinois, April 1993), to be published by the Mathematical Association of America, Fall 1993.

**Appendices**

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# Appendix 1

## 1992 draft of text
## *A Companion to Calculus*

(only one copy submitted)

# Appendix 2

# Evaluation Report of the two-year project

MORAVIAN COLLEGE

FIPSE-funded project, 1991-1993

*Development of course materials to integrate precalculus review with the first course in Calculus*

Evaluation Report

The purpose of this project was to produce and class-test course materials to accompany a slower-paced first course in calculus that integrates the review of algebra, functions and graphing, and problem-solving strategies with the introduction of the new concepts and techniques of calculus. The goals were to increase student understanding, improve morale, and attain a higher rate of completion of the Calculus I course by students who were not prepared to take the traditional one-semester course.

To assess how well we accomplished these goals in the two years of the project, we carried out several different evaluations, each measuring a different aspect. The categories of the evaluations are as follows:

1. Data on course enrollment, attrition, and completion of the Precalculus-Calculus I sequence for the years 1986–1988 (2 years prior to the introduction of the integrated course) and similar data for the Calculus With Review (Math 106–166) course 1988–1993.
pages 2–3

2. Comparative evaluation of student mastery of precalculus material prior to entering the Calculus With Review (Math 106) course and after completing Part I of the course.
pages 4–7

3. Comparative evaluation of student performance on common questions on the final examinations in the Calculus With Review (Math 106–166) course and the standard Calculus I (Math 170) course. There were two separate evaluations: Fall (Part I of Calculus With Review) and Spring (Part II of Calculus With Review).
pages 8–17

4. Survey of student attitudes towards mathematics, the Calculus With Review (Math 106–166) course, and the materials, at the end of each semester.
pages 18–27

We are indebted to Dean William Deeds for his assistance in preparing the student attitude survey and to Dawn Voorhees and Matthew McShea for the compilation of data for all parts of this assessment.

This report was prepared by Doris Schattschneider, Project Director, award # P116B12036.

Data on Attrition: Precalculus-Calculus sequence and Math 106-166 Calculus With Review

Moravian College attrition figures for years 1986 - 1988, two years prior to the introduction of the integrated course until the present.

Precalculus - Calculus I sequence

| | Precalculus Enrollment | Withdrew or Failed | Completed Precalculus | Continued to Calculus I | Completed Calculus I |
|---|---|---|---|---|---|
| 1986-1987 | 110 | 17 | 93 | 64 | 51 |
| 1987-1988 | 92 | 25 | 67 | 40 | 31 |

| | % enrolled who completed Precalculus | % enrolled in Precalculus who continued to Calculus I | % enrolled in Precalculus who completed Calculus I same year |
|---|---|---|---|
| 1986-1987 | 84.5% | 58.2% | 46.4% |
| 1987-1988 | 72.8% | 43.5% | 33.7% |

Math 106-166 Calculus I With Review (CWR) one year course

| | CWR Part I Enrollment | Withdrew or Failed | Completed CWR Part I | Continued to CWR Part II | Completed CWR Part II |
|---|---|---|---|---|---|
| 1988-1989 | 122 | 46 | 76 | 53 | 37 |
| 1989-1990 | 127 | 23 | 104 | 94 | 74 |
| 1990-1991 | 102 | 15 | 87 | 64 | 56 |
| 1991-1992 | 112 | 18 | 94 | 81 | 59 |
| 1992-1993 | 85 | 21 | 64 | 54 | 44 |

| | % enrolled who completed CWR I | % enrolled in CWR I who continued to CRW II | % enrolled in CWR I who completed CWR II same year |
|---|---|---|---|
| 1988-1989 | 62.3% | 43.4% | 30.3% |
| 1989-1990 | 81.9% | 74.0% | 58.3% |
| 1990-1991 | 85.3% | 62.7% | 54.9% |
| 1991-1992 | 83.9% | 72.3% | 52.7% |
| 1992-1993 | 75.3% | 63.5% | 51.8% |

*Comments on attrition figures.*

In the Precalculus-Calculus sequence, a significant number of students who passed Precalculus did not enroll in the Calculus I course. In 1986-1987, 31% of the students did not continue, and in 1988-1989, the percentage was 40%. Some of the drop-off may be attributed to the fact that students could satisfy a mathematics "liberal education guideline" (distribution requirement) by completing just the Precalculus course; there were undoubtedly some who took it without strong intention to continue to Calculus I. The integrated course Calculus I With Review still fulfills the guideline, even if students complete only the first semester, but in this case, students at least have covered the topics of limit and derivative— in short, they have at least some knowledge of calculus, whereas in the past, they saw nothing beyond topics in high school Algebra and Trigonometry. It is likely that fewer students sign up for a course called Calculus without the intent of completing the full year course. In 1988-1989, 30% of those who passed Part I did not continue to Part II; in 1989-1990, 10% did not continue; in 1990-1991, 26% did not continue; in 1991-1992, 14% did not continue; and in 1992-1993, 16% did not continue.

A special note should be made with regard to attrition in the evening class, which was almost entirely adult students. For the 1991-1992 year, all 24 students in the evening class completed Math 106, and 1 student decided not to continue to Math 166. All students who continued completed Math 166; thus there was a 96.2% completion rate of the two-semester course for this group. In 1992-1993, attritrion in this group was comparable to the day sections: 5 students in the Math 106 evening class (with enrollment of 19) withdrew from the course, and one failed it. All but 2 of those who completed Math 106 continued to Math 166, and all but one completed Math 166; thus there was a 53% completion rate of the two-semester course for this group.

Each year, several of the students who do not continue to Math 166 after completing Math 106, or who withdraw from Math 166 in the spring, enroll in Math 166 in spring of the following year. The completion rate for these students is not reflected in the data in the tables.

One unmeasurable variable that sometimes affects attrition is the instructors teaching the course. In each of the two years, different faculty members taught the Math 106–166 course, using the same syllabus and materials (a total of seven different instructors).

One last comparison on attrition: In 1991–1992, 114 students enrolled in the "regular" one-semester Calculus I course (Math 170) and 92 (81%) completed the course. In 1992-1993, 131 students enrolled in Math 170 and 92 (70%) completed the course.

## Pre-and-post comparison of Math 106 freshmen performance on precalculus questions, Fall 1991

Prior to freshman registration, Moravian College administers to all freshmen a high school mathematics assessment test; most take it in May, near the end of their senior year in high school. Thirteen of these questions were also on the final examination for Math 106. Questions were graded uniformly for comparative purposes. Grades for each of the freshmen on each question on each of the two tests were recorded and compared, for the purpose of assessing improvement in understanding. Questions 1–11 were multiple choice, with no partial credit. Questions 12 and 13 had several parts, each receiving credit; a perfect grade for question 12 was 10 and for question 13 was 4. The 13 questions appear on pages 6–7 of this report. There were 55 freshmen who took both tests in 1991.

The following summary indicates for each of the 13 questions, the number of students (of the total of 55) whose answers fall into each category:

C HSA = correct on high school math assessment

C F = correct on Math 106 final

C Both = correct on both the high school math assessment and on the Math 106 final

Inc Both = incorrect on both the high school math assessment and on the Math 106 final

Down = right on high school math assessment and wrong on Math 106 final (or grade worse, questions 12 & 13)

Same = same grade on both the high school math assessment and on the Math 106 final

Up = wrong on high school math assessment and right on Math 106 final (or grade better, questions 12 & 13)

| Quest. | C HSA | C F | C Both | Inc Both | Down | Same | Up |
|---|---|---|---|---|---|---|---|
| 1. | 39 | 52 | 39 | 3 | 0 | 42 | 13 |
| 2. | 26 | 40 | 23 | 12 | 3 | 35 | 17 |
| 3. | 17 | 31 | 14 | 21 | 3 | 35 | 17 |
| 4. | 19 | 27 | 12 | 21 | 7 | 33 | 15 |
| 5. | 5 | 14 | 2 | 38 | 3 | 40 | 12 |
| 6. | 19 | 33 | 12 | 15 | 7 | 27 | 21 |
| 7. | 31 | 41 | 25 | 8 | 6 | 33 | 16 |
| 8. | 10 | 19 | 6 | 18 | 4 | 24 | 13 |
| 9. | 10 | 27 | 4 | 22 | 6 | 26 | 23 |
| 10. | 20 | 46 | 19 | 7 | 1 | 26 | 28 |
| 11. | 23 | 47 | 22 | 7 | 1 | 29 | 25 |

| | GRADE HSA | | | | | | GRADE FINAL | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 2 | 4 | 6 | 8 | 10 | 0 | 2 | 4 | 6 | 8 | 10 | | | |
| 12. | 11 | 23 | 13 | 4 | 4 | 0 | 5 | 6 | 11 | 11 | 11 | 11 | 7 | 9 | 39 |
| | 0 | 1 | 2 | 3 | 4 | | 0 | 1 | 2 | 3 | 4 | | | | |
| 13. | 7 | 13 | 11 | 12 | 12 | | 1 | 3 | 14 | 15 | 22 | | 9 | 14 | 31 |

## Pre-and-post comparison of Math 106 freshmen performance on precalculus questions, Fall 1992

The same procedure was carried out in Fall 1992 as in Fall 1991 to assess the improvement in understanding of precalculus concepts. For uniformity in comparison, the same pretest (the high school assessment) was taken prior to enrollment in the Math 106 course, and the same Math 106 final examination was given at the end of the semester. Grades for the same 13 questions common to both examinations were compared and results tabulated as on the 1991 report. The 13 questions appear here on pages 6–7. There were 42 freshman who took both tests in 1992.

The following summary indicates for each of the 13 questions, the number of students (of the total of 42) whose answers fall into each category:

C HSA = correct on high school math assessment

C F = correct on Math 106 final

C Both = correct on both the high school math assessment and on the Math 106 final

Inc Both = incorrect on both the high school math assessment and on the Math 106 final

Down = right on high school math assessment and wrong on Math 106 final (or grade worse, questions 12 & 13)

Same = same grade on both the high school math assessment and on the Math 106 final

Up = wrong on high school math assessment and right on Math 106 final (or grade better, questions 12 & 13)

| Quest. | C HSA | C F | C Both | Inc Both | Down | Same | Up |
|---|---|---|---|---|---|---|---|
| 1. | 27 | 36 | 25 | 4 | 2 | 29 | 11 |
| 2. | 23 | 34 | 22 | 7 | 1 | 29 | 12 |
| 3. | 8 | 21 | 5 | 18 | 3 | 23 | 16 |
| 4. | 12 | 15 | 5 | 20 | 7 | 25 | 10 |
| 5. | 6 | 18 | 3 | 21 | 3 | 24 | 15 |
| 6. | 12 | 21 | 8 | 17 | 4 | 25 | 13 |
| 7. | 26 | 33 | 21 | 4 | 5 | 25 | 12 |
| 8. | 13 | 25 | 11 | 15 | 2 | 26 | 14 |
| 9. | 11 | 18 | 7 | 20 | 4 | 27 | 11 |
| 10. | 18 | 29 | 10 | 5 | 8 | 15 | 19 |
| 11. | 21 | 36 | 20 | 5 | 1 | 25 | 16 |

| | GRADE HSA | | | | | | GRADE FINAL | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 2 | 4 | 6 | 8 | 10 | 0 | 2 | 4 | 6 | 8 | 10 | | | |
| 12. | 6 | 14 | 11 | 7 | 3 | 1 | 5 | 2 | 8 | 7 | 21 | 4 | 1 | 6 | 35 |
| | 0 | 1 | 2 | 3 | 4 | | 0 | 1 | 2 | 3 | 4 | | | | |
| 13. | 2 | 10 | 15 | 4 | 11 | | 2 | 3 | 3 | 10 | 24 | | 7 | 11 | 24 |

The figures make clear that in both 1991 and 1992, for every question, there was improvement in the numbers answering correctly. The most dramatic improvement in both years was for the graphing questions 12 and 13.

Questions 1–13, common to the high school math assessment and the final exam in Math 106.

1. $(3n - 2)^2 = ?$

a. $9n^2 - 4$ b. $9n^2 + 4$ c. $9n^2 - 12n + 4$ d. $9n^2 - 6n + 4$

2. $(x + y)^{-1} = ?$

a. $-1.(x + y)$ b. $x^{-1} + y^{-1}$ c. $\frac{1}{x + y}$ d. none of these

3. $\frac{\frac{1}{x}}{5x + \frac{1}{x}} = ?$

a. $\frac{1}{5x + 1}$ b. $\frac{1}{5 + \frac{1}{x^2}}$ c. $\frac{x}{5x^2 + 1}$ d. $\frac{1}{5x^2 + 1}$

4. $(3x^{1/4})^2 = ?$

a. $3\sqrt{x}$ b. $9\sqrt{x}$ c. $\sqrt{3x}$ d. $(3x^2)^{1/4}$

5. If $f(w) = 3w^3 + 7w^{-1}$, then $f(-2) = ?$

a. $\frac{-55}{2}$ b. $\frac{-41}{2}$ c. $\frac{41}{2}$ d. $\frac{55}{2}$

6. The values of x for which $(x - 1)(x + 2) < 0$ are

a. $x > 1$ or $x < -2$ b. $-2 < x < 1$ c. $x \leq -2$ d. when x is negative.

7. If $3x + 5 < -x - 3$, then

a. $x > 2$ b. $x < 2$ c. $x > -2$ d. $x < -2$

8. If $|x - 1| < 5$, then

a. $-5 < x < 5$ b. $x < 6$ c. $-4 < x < 6$ d. none of these.

9. $(t^{5/2})(t^{-3}) = ?$ a. $\frac{1}{t}$ b. is negative c. $\frac{1}{t^7\sqrt{t}}$ d. $\frac{1}{\sqrt{t}}$

10. Multiply out and simplify; circle the correct answer.

$$(\sqrt{x + 1} - \sqrt{x})(\sqrt{x + 1} + \sqrt{x})$$

a. 1 b. $2x + 1$ c. x d. $2\sqrt{x + 1}\sqrt{x}$

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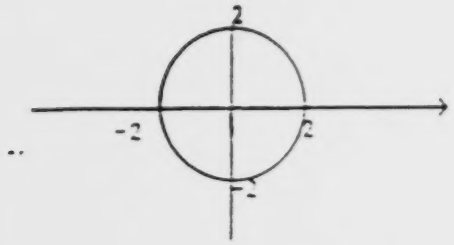
11. If $f(t) = 3t^2 + 7t$, then $f(x+h) = ?$

a. $3x^2 + 7x + h$ b. $3(x^2 + h^2) + 7(x + h)$

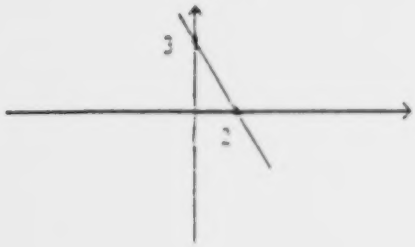
c. $3x^2 + 7x + 3h^2 + 7h$ d. $3(x+h)^2 + 7(x+h)$

12. Six graphs are shown below, and the correct equations for the graphs can be found among those listed at the bottom. For each graph, write its equation under the graph.

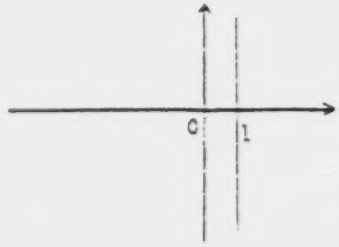
a.




b.

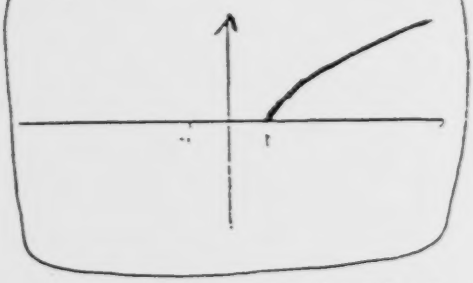



c.

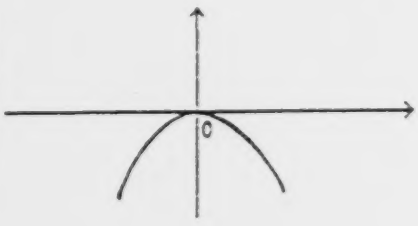



not counted for purposes of comparison with actual assessment
different graph (conic) on this

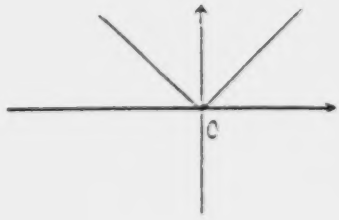
d.




e.




f.




| | | | | |
|---|---|---|---|---|
| $y = 1$ | $x^2 + y^2 = 2$ | $y = \sqrt{x-1}$ | $y = 4x^2$ | $2x + 3y = 6$ |
| $2y + 3x = 6$ | $y = \sqrt{x+1}$ | $y = -4x^2$ | $x^2 + y^2 = 4$ | $x = \lvert y \rvert$ |
| $y = x$ | $\lvert x \rvert = \lvert y \rvert$ | $x = -4y^2$ | $x = 1$ | $y = \lvert x \rvert$ |

13. Which of the graphs in number 12 above are graphs of functions? Circle the corresponding letters below.

a b c d e f

## Comparison of Math 106 (Calculus with Review) and Math 170 (one-semester calculus) students' performance on common questions on final examinations, Fall 1991.

In order to compare the degree of mathematical understanding of students in the Math 106 course with those in the 170 course, 12 questions that tested calculus material common to both courses were on the final examinations for both courses. The 12 questions are on pages 12–13 of this report. The table below summarizes the mean score, median score, and standard deviation on each of the 12 questions for the two groups. In 1991, there were 98 students (in 4 sections) who took the Math 106 final exam and 78 students (in 4 sections) who took the Math 170 final exam.

| Question | Score possible | 106 mean | 170 mean | 106 median | 170 median | 106 st. dev. | 170 st. dev. |
|---|---|---|---|---|---|---|---|
| 1. | 4 | 3.286 | 3.423 | 4 | 4 | 1.332 | 1.254 |
| 2. | 4 | 1.806 | 2.583 | 2 | 2 | 1.544 | 1.385 |
| 3. | 4 | 2.643 | 2.846 | 4 | 4 | 1.868 | 1.721 |
| 4. | 12 | 8.786 | 9.795 | 10 | 12 | 2.905 | 3.143 |
| 5. | 3 | 1.879 | 2.167 | 2 | 3 | 1.239 | 1.178 |
| 6. | 3 | 1.597 | 2.071 | 2 | 2.5 | 1.198 | 1.104 |
| 7. | 6 | 3.041 | 5.013 | 3 | 5.5 | 2.153 | 1.398 |
| 8. | 6 | 3.469 | 5.359 | 4.5 | 6 | 2.5 | 1.316 |
| 9. | 8 | 4.898 | 6.237 | 6 | 8 | 2.965 | 2.581 |
| 10. | 8 | 3.398 | 5.788 | 3 | 7 | 2.849 | 2.475 |
| 11. | 11 | 5.010 | 6.462 | 5 | 6.5 | 3.257 | 3.136 |
| 12. | 13 | 7.423 | 8.282 | 8 | 8.5 | 2.493 | 2.967 |

On all questions, the Math 170 students had a higher mean score than the Math 106 students. However, many of these differences were slight, and for questions 1, 2, and 3, the median score was the same for both groups. The two groups differed most greatly in their scores on questions 7 and 10.

It should be noted that the Math 106 population of 98 students consisted of two distinct groups: "traditional" students, including 55 freshmen right from high school, and approximately 25 "adult" students (in the evening course), all of whom had not studied mathematics for periods of 3 – 20 years. On all but 3 questions, the performance of the evening section was markedly better than the other 106 sections, and on questions 1, 11, and 12, they outperformed the 170 students. The chart below summarizes their mean scores on the 12 common questions; compare this data with that in the two columns "106 mean" and "170 mean" above.

106 evening section mean scores on questions 1.–12.

| Question | mean score | Question | mean score | Question | mean score |
|---|---|---|---|---|---|
| 1. | 3.925 | 5. | 1.87 | 9. | 5.148 |
| 2. | 1.63 | 6. | 1.703 | 10. | 4.85 |
| 3. | 2.407 | 7. | 3.926 | 11. | 7.593 |
| 4. | 9.37 | 8. | 4.185 | 12. | 8.722 |



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## Comparison of Math 106 (Calculus with Review) and Math 170 (one-semester calculus) students' performance on common questions on final examinations, Fall 1992.

The same procedure was carried out in Fall 1992 as in Fall 1991 to compare the degree of mathematical understanding of students in the Math 106 course with those in the 170 course. For uniformity in comparison, the common questions on the Math 106 final examination and Math 170 final examination were the same as those in 1991. Grades for these 12 questions common to both examinations were compared and results tabulated as on the 1991 report. The 12 questions appear on pages 12–13 of this report. In 1992, there were 71 students (in 4 sections) who took the 106 final exam and 85 students (in 4 sections) who took the 170 final exam .

| Question | Score possible | 106 mean | 170 mean | 106 median | 170 median | 106 st. dev. | 170 st. dev. |
|---|---|---|---|---|---|---|---|
| 1. | 4 | 3.155 | 3.271 | 4 | 4 | 1.546 | 1.366 |
| 2. | 4 | 2.127 | 2.800 | 2 | 3 | 1.715 | 1.462 |
| 3. | 4 | 2.662 | 2.976 | 4 | 4 | 1.874 | 1.640 |
| 4. | 12 | 5.775 | 8.812 | 5 | 10 | 4.495 | 3.743 |
| 5. | 3 | 1.592 | 2.235 | 2 | 3 | 1.400 | 1.161 |
| 6. | 3 | 1.408 | 1.753 | 1 | 2 | 1.116 | 1.133 |
| 7. | 6 | 4.056 | 4.929 | 5 | 5 | 1.897 | 1.325 |
| 8. | 6 | 3.944 | 5.282 | 5 | 6 | 2.311 | 1.385 |
| 9. | 8 | 4.099 | 5.812 | 4 | 8 | 3.158 | 3.096 |
| 10. | 8 | 3.507 | 6.200 | 3 | 7 | 2.878 | 2.192 |
| 11. | 11 | 4.451 | 5.835 | 5 | 6 | 2.431 | 2.890 |
| 12. | 13 | 6.211 | 8.235 | 6 | 9 | 2.918 | 2.698 |

On all questions, the Math 170 students had a higher mean score than the Math 106 students. However, many of these differences were slight, and for questions 1, 3, and 7, the median score was the same for both groups. The two groups differed most greatly in their scores on questions 4, 10 and 12.

It should be noted that the Math 106 population of 71 students consisted of two distinct groups: "traditional" students, including 42 freshmen right from high school, and approximately 18 "adult" students (in the evening course), all of whom had not studied mathematics for periods of 3 – 20 years. On all but 2 questions, the performance of the evening section was markedly better than the other 106 sections, and on half the questions, they outperformed the 170 students. The chart below summarizes their mean scores on the 12 common questions; compare this data with that in the two columns "106 mean" and "170 mean" above.

106 evening section mean scores on questions 1.–12.

| Question | mean score | Question | mean score | Question | mean score |
|---|---|---|---|---|---|
| 1. | 4.000 | 5. | 1.538 | 9. | 5.000 |
| 2. | 3.385 | 6. | 1.077 | 10. | 5.615 |
| 3. | 3.385 | 7. | 5.231 | 11. | 6.769 |
| 4. | 7.15 | 8. | 4.923 | 12. | 8.692 |

Additional comparisons of Math 106 and Math 170 students.

Indicators for course completion (tables on next page)

The Mathematics Department at Moravian College recommends to freshmen who have not had a calculus course in high school and who will need calculus for their proposed major, which of the two courses—Math 106 or Math 170—to take. The evaluation is based on their performance on the high school math assessment (HSA) administered by the department, their SAT scores, their high school rank RANK (reported as a percent), and their high school math course grades. For Fall 1991, the mean score on the HSA for those registered in Math 106 was 30.78% and the mean score for those registered in Math 170 was 56.28%. For Fall 1992, the mean score on the HSA for those registered in Math 106 was 32.6% and the mean score for those registered in Math 170 was 55.8%. There is no precise formula for the recommendations, and sometimes the various measures give conflicting information. (Students are allowed to move from Math 170 to Math 106 during the first four weeks of class if they request it.) As part of the data analysis for the project, we tried to see which, if any, of these indicators is more reliable as a predictor for staying in the course.

The tables on the next page indicate number (N) of students in the population for which information was available, the number (N*) for which information was missing, MEAN, MEDIAN, 5% trimmed mean (TRMEAN), standard deviation (STDEV), and standard error of the mean (SEMEAN) for each of two groups: those who stayed in the course (STAY), and those who dropped it (DROP). It would appear that for Math 106 students, the only predictor is the HSA score. For the Math 170 population, the high school rank, verbal SAT scores, as well as the HSA score are all significantly lower for those who dropped. However, this observation should be tempered by the fact that only a very small number of students dropped Math 170.

Indicators for grades in calculus

A correlation of HSA grade, high school rank, and course grade for Math 106 and Math 170 showed that there was significant negative correleation between high school rank and course grade, and significant positive correlation between HSA score and course grade. (High school rank as a percentile is low for high-ranking students; number 1 in a class of 100 would have RANK reported as 0.01.)

Correlation for 1991 Math 106

| | Course grade | HS Rank |
|---|---|---|
| HS Rank | -0.421 | |
| HSA grade | 0.386 | -0.182 |

Correlation for 1991 Math 170

| | Course grade | HS Rank |
|---|---|---|
| HS Rank | -0.272 | |
| HSA grade | 0.494 | -0.032 |

Correlation for 1992 Math 106

| | Course grade | HS Rank |
|---|---|---|
| HS Rank | -0.212 | |
| HSA grade | 0.478 | -0.338 |

Correlation for 1992 Math 170

| | Course grade | HS Rank |
|---|---|---|
| HS Rank | -0.340 | |
| HSA grade | 0.473 | -0.269 |

Calculus the first or the second time

The Math 170 population also is composed of two different groups (mixed in each section): those who have not had a calculus course in high school and those who have had a calculus course in high school, but did not qualify for placement in Calculus II (the second semester of calculus). In 1991, the mean and median course grade (final grade) in Math 170 for those who had not had a high school calculus course was 1.63 (C-) and 1.67 (C-), and the mean and median course grade for the others was 2.733 (B-) and 3.0 (B). In 1992, the mean and median course grade (final grade) in Math 170 for those who had not had a high school calculus course was 1.81 (C-) and 2.0 (C), and the mean and median course grade for the others was 2.43 (C+) and 2.33 (C+).

1991 Math 106

| | N | N* | MEAN | MEDIAN | TRMEAN | STDEV | SEMEAN |
|---|---|---|---|---|---|---|---|
| RANKSTAY | 52 | 46 | 40.26 | 39.92 | 39.88 | 17.12 | 2.37 |
| RANKDROP | 10 | 3 | 48.59 | 45.39 | 48.33 | 19.24 | 6.08 |
| SATMSTAY | 60 | 38 | 498.83 | 490.00 | 495.74 | 61.54 | 7.94 |
| SATMDROP | 10 | 3 | 499.00 | 500.00 | 497.50 | 66.90 | 21.20 |
| SATVSTAY | 60 | 38 | 435.50 | 420.00 | 430.40 | 77.90 | 10.10 |
| SATVDROP | 10 | 3 | 470.00 | 480.00 | 472.50 | 62.40 | 19.70 |
| HSA STAY | 57 | 41 | 31.91 | 32.00 | 31.55 | 13.06 | 1.73 |
| HSA DROP | 11 | 2 | 24.91 | 25.00 | 24.78 | 6.16 | 1.86 |

1991 Math 170

| | N | N* | MEAN | MEDIAN | TRMEAN | STDEV | SEMEAN |
|---|---|---|---|---|---|---|---|
| RANKSTAY | 63 | 15 | 23.18 | 19.33 | 20.80 | 21.76 | 2.74 |
| RANKDROP | 6 | 8 | 36.70 | 25.20 | 36.70 | 26.40 | 10.80 |
| SATMSTAY | 64 | 14 | 568.65 | 570.00 | 567.76 | 64.75 | 8.09 |
| SATMDROP | 7 | 7 | 528.60 | 550.00 | 528.60 | 47.10 | 17.80 |
| SATVSTAY | 64 | 14 | 504.22 | 490.00 | 501.55 | 77.19 | 9.65 |
| SATVDROP | 7 | 7 | 425.70 | 430.00 | 425.70 | 33.60 | 12.70 |
| HSA STAY | 65 | 13 | 57.54 | 59.00 | 57.69 | 16.30 | 2.02 |
| HSA DROP | 7 | 7 | 44.57 | 46.00 | 44.65 | 18.27 | 6.90 |

1992 Math 106

| | N | N* | MEAN | MEDIAN | TRMEAN | STDEV | SEMEAN |
|---|---|---|---|---|---|---|---|
| RANKSTAY | 42 | 16 | 40.52 | 34.26 | 39.45 | 23.17 | 3.58 |
| RANKDROP | 5 | 8 | 40.00 | 50.30 | 40.00 | 24.70 | 11.10 |
| SATMSTAY | 46 | 12 | 486.30 | 490.00 | 483.80 | 68.70 | 10.10 |
| SATMDROP | 5 | 8 | 480.00 | 510.00 | 480.00 | 92.50 | 11.40 |
| SATVSTAY | 46 | 12 | 437.61 | 430.00 | 435.24 | 64.16 | 9.46 |
| SATVDROP | 5 | 8 | 396.00 | 400.00 | 396.00 | 70.90 | 31.70 |
| HSA STAY | 44 | 14 | 34.02 | 32.00 | 33.92 | 13.30 | 2.01 |
| HSA DROP | 6 | 7 | 22.17 | 20.50 | 22.17 | 6.62 | 2.70 |

1992 Math 170

| | N | N* | MEAN | MEDIAN | TRMEAN | STDEV | SEMEAN |
|---|---|---|---|---|---|---|---|
| RANKSTAY | 67 | 18 | 20.22 | 15.62 | 19.16 | 15.18 | 1.85 |
| RANKDROP | 20 | 4 | 35.01 | 38.71 | 34.88 | 21.21 | 4.74 |
| SATMSTAY | 71 | 14 | 569.01 | 570.00 | 570.00 | 71.44 | 8.48 |
| SATMDROP | 20 | 4 | 554.00 | 565.00 | 555.60 | 73.20 | 16.40 |
| SATVSTAY | 71 | 14 | 467.89 | 470.00 | 467.78 | 75.76 | 8.99 |
| SATVDROP | 20 | 4 | 447.00 | 435.00 | 445.60 | 73.70 | 16.50 |
| HSA STAY | 38 | 47 | 59.32 | 60.0 | 59.65 | 16.06 | 2.61 |
| HSA DROP | 19 | 5 | 48.84 | 48.00 | 48.88 | 10.13 | 2.32 |

Questions 1. –12., common to the Math 106 and Math 170 final examinations 1991–1992.

1.– 3. Find the following limits. Show your computation. If the limit is an infinite limit, say whether it is $\infty$ or $-\infty$. If you claim that the requested limit does not exist, then explain why not.

1. $\lim_{x \to 3} \dfrac{x^2 - x - 6}{x^2 - 3x}$

2. $\lim_{x \to -2^+} \dfrac{x^2 + 3x}{x + 2}$

3. $\lim_{x \to \infty} \dfrac{x^3 + 2x^2 - 4x}{6x^3 - 8x + 10}$

4.–6. Let $f(x) = \begin{cases} 1 - x^2 & \text{if } x < -1 \\ -x^2 & \text{if } -1 \le x < 2 \\ 2x^2 - 12 & \text{if } 2 \le x \end{cases}$

4. Find :

$\lim_{x \to -1^-} f(x)$ $\qquad$ $\lim_{x \to -1^+} f(x)$

$\lim_{x \to -1} f(x)$

$\lim_{x \to 2^-} f(x)$ $\qquad$ $\lim_{x \to 2^+} f(x)$

$\lim_{x \to 2} f(x)$

5. Is the function $f(x)$ continuous at $x = -1$ ? Justify your answer by using the definition of continuity.

6. Is the function $f(x)$ continuous at $x = 2$ ? Justify your answer by using the definition of continuity.

7.–8. Find the derivative of each of the following functions.

7. $y = 2x^5 - \dfrac{4}{x^3} + \dfrac{1}{3\sqrt{x}} + 17$

8. $y = \dfrac{x^2 - 1}{7x + \sqrt[3]{x}}$

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9. Let $f(x) = 2x^2 - 3x$. Use the definition of the derivative to compute the derivative $f'(x)$.

10. Find the equation of the tangent line to the graph of $y = \sqrt{x} + 3$ at the point where $x = 4$.

11. Let $f(x) = \dfrac{3x - 6}{x^2 - 4}$.

    a. Give the domain of $f(x)$.

    b. Give the equations of all horizontal asymptotes.

    c. Why are these horizontal asymptotes?

    d. Give the equations of all vertical asymptotes.

    e. Why are these vertical asymptotes?

12. The graph of $y = f(x)$ is given below.

a. $\lim_{x \to 0^+} f(x) =$ _____

b. $\lim_{x \to 0^-} f(x) =$ _____

c. $\lim_{x \to -\infty} f(x) =$ _____

d. $\lim_{x \to -1^-} f(x) =$ _____

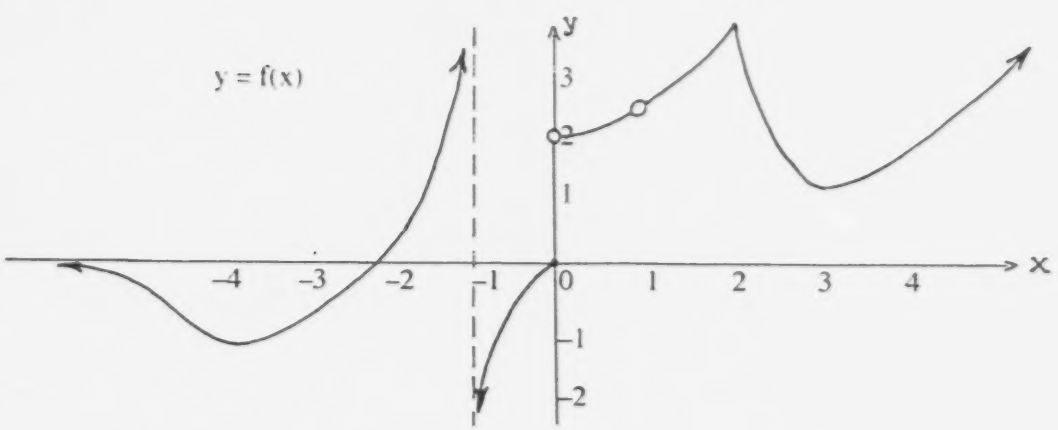
e. List each value of "a" where $\lim_{x \to a} f(x)$ does not exist. _____

f. List each x value where f(x) is discontinuous.

g. $f'(x) = 0$ for $x =$ _____.

h. Give the equations of any vertical asymptotes. _____

i. Give the equations of any horizontal asymptotes. _____



## Comparison of Math 166 (Calculus with Review Part II) and Math 170 (one-semester calculus) students' performance on common questions on final examinations, Spring 1992.

In order to compare the degree of mathematical understanding of students in the Math 166 course with those in the 170 course, 13 questions that tested calculus material common to both courses were on the final examinations for both courses. The 13 questions are on pages 16–17 of this report. The table below summarizes the mean score, median score, and standard deviation on each of the 13 questions for the two groups. There were 69 students who took the 166 final exam (in 3 sections) and 98 students who took the 170 final exam (in 5 sections: 4 day sections Fall 1991 and 1 evening section, Spring 1992).

| | Score | | | | | | |
|---|---|---|---|---|---|---|---|
| Question | possible | 166 mean | 170 mean | 166 median | 170 median | 166 st. dev. | 170 st. dev. |
| 1. | 6 | 4.0 | 4.612 | 4.0 | 6.0 | 1.964 | 1.887 |
| 2. | 6 | 2.153 | 3.184 | 2.0 | 3.0 | 2.134 | 2.438 |
| 3. | 6 | 2.861 | 3.786 | 3.0 | 5.0 | 2.596 | 2.442 |
| 4. | 8 | 5.514 | 6.163 | 6.0 | 7.5 | 2.478 | 2.535 |
| 5. | 8 | 6.375 | 5.459 | 7.0 | 6.0 | 1.772 | 2.36 |
| 6. | 6 | 4.069 | 4.281 | 5.0 | 6.0 | 2.254 | 2.345 |
| 7. | 5 | 2.917 | 3.592 | 4.0 | 4.0 | 2.154 | 1.845 |
| 8. | 4 | 1.729 | 2.066 | 2.0 | 2.75 | 1.547 | 1.774 |
| 9. | 6 | 1.806 | 2.622 | 0.5 | 3.0 | 2.237 | 0.234 |
| 10. | 8 | 2.083 | 3.847 | 0.0 | 4.0 | 3.001 | 3.285 |
| 11. | 22 | 13.174 | 14.046 | 14.5 | 14.5 | 6.391 | 7.207 |
| 12. | 11 | 5.847 | 6.245 | 6.0 | 6.0 | 1.964 | 2.433 |
| 13. | 8 | 3.361 | 4.102 | 2.0 | 4.0 | 3.502 | 3.498 |

On all questions but one, the Math 170 students had a higher mean score than the Math 166 students. On question 5, both mean and median score were higher for Math 166 students, and on questions 7 and 12, the median score was the same for both groups. The two groups differed most greatly in their scores on question 10.

## Comparison of Math 166 (Calculus with Review Part II) and Math 170 (one-semester calculus) students' performance on common questions on final examinations, Spring 1993.

The same procedure was carried out in Spring 1993 as in Spring 1992 to compare the degree of mathematical understanding of students in the Math 166 course with those in the 170 course. For uniformity in comparison, the common questions on the Math 166 final examination and Math 170 final examination were, with one exception, the same as those in 1992. Question 10 was omitted from the 1993 comparison. Grades for the remaining 12 questions common to both examinations were compared and results tabulated as on the 1992 report. (The 13 questions appear pages 16–17.)

In 1993, this comparison data was not available for the evening section of 166, so that the table below represents scores of 35 students (in 2 sections) who took the 166 final exam and 99 students who took the 170 final exam (5 sections: 4 day sections Fall 1992 and 1 evening section, Spring 1993). Since questions 1 and 3 had to be changed for the evening 170 section, the 170 data below for those two questions is just for the 85 students in 4 day sections.

| Question | Score possible | 166 mean | 170 mean | 166 median | 170 median | 166 st. dev. | 170 st. dev. |
|---|---|---|---|---|---|---|---|
| 1. | 6 | 2.314 | 4.412 | 2.0 | 5.0 | 2.435 | 1.755 |
| 2. | 6 | 0.600 | 3.414 | 0.0 | 3.0 | 1.418 | 2.272 |
| 3. | 6 | 1.971 | 3.624 | 0.0 | 4.0 | 2.662 | 2.370 |
| 4. | 8 | 5.486 | 6.24 | 6.0 | 7.0 | 2.406 | 2.195 |
| 5. | 8 | 4.543 | 5.505 | 4.0 | 6.0 | 2.973 | 2.182 |
| 6. | 6 | 3.571 | 4.525 | 5.0 | 5.0 | 2.512 | 2.022 |
| 7. | 5 | 0.886 | 3.667 | 0.0 | 5.0 | 1.922 | 1.852 |
| 8. | 4 | 2.829 | 2.808 | 4.0 | 3.0 | 2.149 | 1.448 |
| 9. | 6 | 1.514 | 2.333 | 0.0 | 2.0 | 1.991 | 2.395 |
| 10. | omitted | | | | | | |
| 11. | 22 | 13.23 | 14.939 | 15.0 | 16.0 | 6.53 | 6.454 |
| 12. | 11 | 5.20 | 6.556 | 5.0 | 7.0 | 1.762 | 2.396 |
| 13. | 8 | 1.714 | 4.020 | 0.0 | 4.0 | 3.11 | 2.390 |

On all questions but one (question 8), the mean scores of the Math 166 students were lower than the Math 170 students, and on 4 questions, they were significantly lower (2 points or more lower). On questions 2, 3, 7, and 13, the Math 166 scores were much lower than in 1992, while the Math 170 scores were about the same.

Questions 1.–13.. common to the Math 166 and Math 170 final examinations 1991–1992

1.&2. Find the derivatives of each of the following functions.

1. $f(x) = e^{5x}(\tan(x^2))$

2. $y = \sin^4(3x^2 + 1)$

3. Differentiate the function $\ln\left(\dfrac{3x+1}{2-x}\right)$

4. Use implicit differentiation to find the derivative dy/dx if

$$x^2 + 2x^3y - y^2 = \cos x$$

5. Find the absolute maximum and the absolute minimum values of the function $f(x) = x^3 - 12x$ on the interval $[-1, 3]$.

6. Find an antiderivative G(x) of the following function g(x):

$$g(x) = 4x^5 - 2\sqrt[3]{x} + \frac{6}{x^3}$$

7. Evaluate the following definite integrals:

$$\int_1^2 x^3 - 4x + 3 \; dx$$

8. $$\int_{-\pi/2}^{\pi} \cos x \; dx$$

9. Given the derivative $f'(x)$ below, find $f(x)$.

$$f'(x) = \frac{4x^3 + 2x^2 - 3}{x}$$

10. Find the area under the curve $y = e^x$ over the interval $[0, \ln 2]$.

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11. The function $f(x) = x^3 - 3x^2$ has derivatives $f'(x) = 3x^2 - 6x$ and $f''(x) = 6x - 6$. Answer all questions below with regard to this function.

3 (a) Find all intercepts of $f(x)$.

4 (b) Find all the intervals on which $f(x)$ is increasing and all the intervals on which $f(x)$ is decreasing. (State which is which!)

2 (c) Find and identify all local maxima.

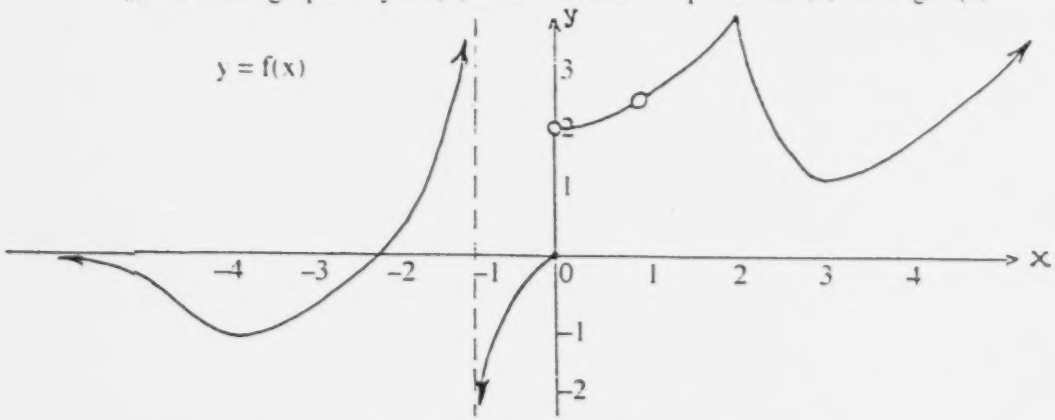
2 (d) Find and identify all local minima.

4 (e) Find all the intervals on which $f(x)$ is concave up and all the intervals on which $f(x)$ is concave down. (State which is which!)

2 (f) Find all points of inflection.

5 (g) Using all the information obtained in (a) through (f) above, carefully sketch the graph of $f(x)$ on the axes below. (Integer values on the axes are indicated by small circles.)

12. Use the graph of $y = f(x)$ below to answer questions (a) through (e).




(a) List each value of $x$ where $f(x)$ is discontinuous. _____

(b) $f'(x) = 0$ for $x =$ _____.

(c) List each value of $f(x)$ where $f(x)$ is not differentiable. _____

(d) The absolute minimum value of $f(x)$ on $[2, \infty]$ is _____.

(e) $f(x)$ has local maximum value(s) when $x =$ _____.

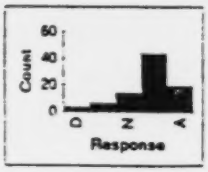
13. A stone is dropped into a still pond causing circular ripples. If the radius of one of the ripples is increasing at a rate of 16 cm/s, at what rate is the area inside this ripple increasing when its radius is 4 cm? (The area of a circle is $A = \pi r^2$.)

## Student Evaluation of Calculus With Review, Part I (Math 106), Fall 1991

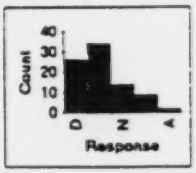
**Number of respondents: 85**

**10 questions on attitudes, 5 questions on use of text materials (histograms of responses below), 3 questions on understanding (responses on page 20). For questions 1–10, D = disagree, N = neutral, and A = agree; for questions 11–15, D = never, N = half the time, and A = always.**

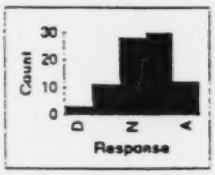
1. This course has improved my ability to solve mathematical problems.



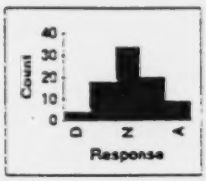

2. I am no better at mathematics than when I began this course.



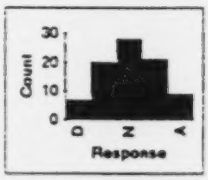

3. Because of this course, I feel able to take further courses in mathematics.



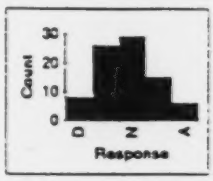

4. Because of this course, I get less anxious about mathematical problems.



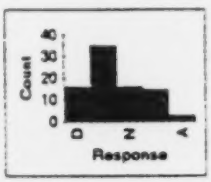

5. Mathematics is just an obstacle to overcome in my college studies.



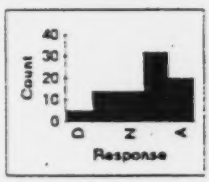

6. Thinking about mathematical problems causes me stress.



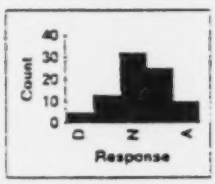

7. Learning mathematics just involves memorizing a set of specific techniques.



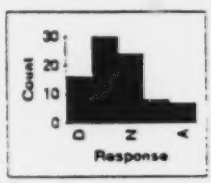

8. The material in this course will be useful to me as I complete my major.



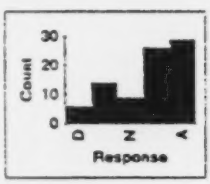

9. Because of this course, I am more aware of the use of mathematics in interpreting the world around me.



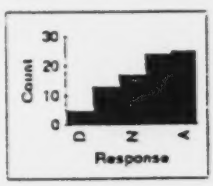

10. I see no need to ever use the material covered in this course again.



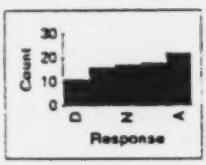

11. Of the assigned readings in this course, I read the handout material.




12. Of the assigned readings in this course, I read the Stewart *Calculus* (textbook).




13. I found the handout materials helpful in understanding the course content.

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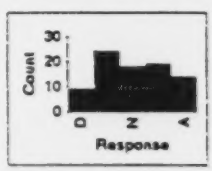
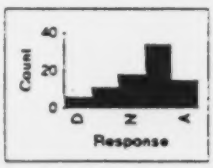



15. I was able to complete my homework assignments.

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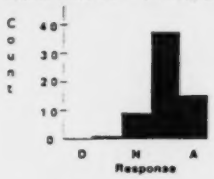
## Student Evaluation of Calculus With Review, Part I (Math 106), Fall 1992

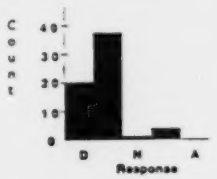
Number of respondents: 62

10 questions on attitudes, 5 questions on use of text materials (histograms of responses below), 3 questions on understanding (responses on page 21). For questions 1–10, D = disagree, N = neutral, and A = agree; for questions 11–15, D = never, N = half the time. and A = always.

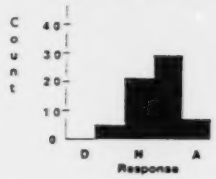
1. This course has improved my ability to solve mathematical problems.



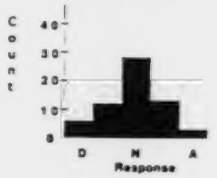

2. I am no better at mathematics than when I began this course.



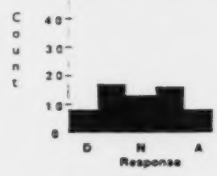

3. Because of this course, I feel able to take further courses in mathematics.



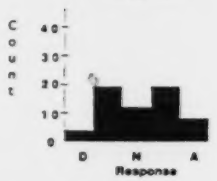

4. Because of this course, I get less anxious about mathematical problems.



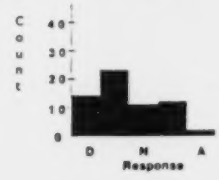

5. Mathematics is just an obstacle to overcome in my college studies.



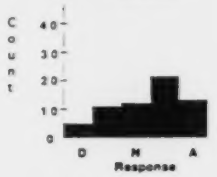

6. Thinking about mathematical problems causes me stress.



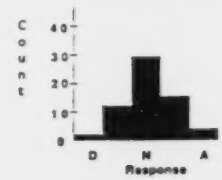

7. Learning mathematics just involves memorizing a set of specific techniques.



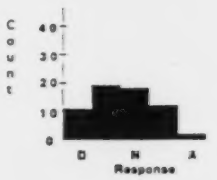

8. The material in this course will be useful to me as I complete my major.



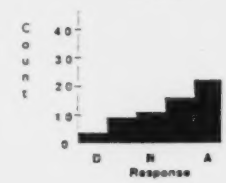

9. Because of this course, I am more aware of the use of mathematics in interpreting the world around me.



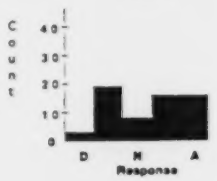

10. I see no need to ever use the material covered in this course again.



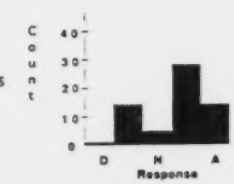

11. Of the assigned readings in this course, I read the handout material




12. Of the assigned readings in this course, I read the Stewart *Calculus* (textbook)




13. I found the handout materials helpful in understanding the course content.

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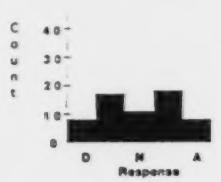
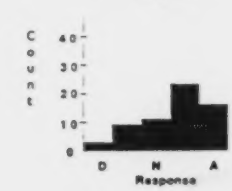



15. I was able to complete my homework assignments.



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## Summary of student responses to questions on understanding, Math 106, Fall 1991

Responses are given by expected course grade. There were 85 respondents.

1. List three major topics covered in this course.
*Each letter represents 2 responses; letters in parantheses are each 1 response*

| | | |
|---|---|---|
| Derivatives: | A A A A (AB) B B B B B B B B B B (BC) C C C C C C C C C C C C C D D D D D | 70 |
| Limits: | A A A A B B B B B B B B (BC) C C C C C C C C C C C D D D D | 55 |
| Functions: | A A A (AB) B B B B B B (BC) C C C C C C (CD) D D | 39 |
| Graphing: | A B B (BC) C D D | 13 |
| Inequalities: | A (BC) C D D | 9 |
| No response: | A (AB) B (BC) D | 9 |
| Equations: | B B (BC) C D | 9 |
| Continuity: | B B C D | 7 |
| Algebra: | (AB) (BC) D | 6 |
| Rate of Chg: | (AB) C C | 5 |
| Asymptotes: | B C | 3 |

Others 1 response: word problems, vectors, factoring, polynomials, formulas, trignometry, geometry, math, humpties.

2. Describe how this course has changed or not changed what you think mathematics is.

| | | |
|---|---|---|
| Hasn't changed: | A A A A A B B B B B B B B B B B C C C C C D D D D D | 25 |
| No response: | A A A A A B B B B B B B B B B B C C C C C D D | 22 |
| Useful in life: | B B B B B C C C | 8 |
| Better understanding: | B B B B C C C | 7 |
| Broader outlook: | C C C C C D | 6 |
| Necessary: | C D D | 3 |
| Not impossible: | A C | 2 |
| Not useful: | C C | 2 |
| Afraid: | C C | 2 |

Others 1 response: All basic logic, undestanding more important than memorizing, still like, changed, difficult, hate it, self-taught subject, respect for subject

3. Describe how this course has changed or not changed how you <u>feel</u> about mathematics.

| | | |
|---|---|---|
| No response: | A A A A B B B B B B B C C C D | 15 |
| More confident: | A A B B B B B B B C C | 11 |
| Still like: | A A A A B B B B C C | 10 |
| Hasn't changed: | A B B C C C D D D D | 10 |
| Bettter understand | B B B B C D D | 7 |
| Too difficult: | C C C D D D D | 7 |
| Useful in life: | B B B C C | 5 |
| Don't like: | B C C | 3 |
| Less difficult: | C C C | 3 |
| Afraid: | C C C | 3 |
| Not useful: | A B | 2 |
| Less confident: | B B | 2 |
| More comfortable: | C C | 2 |
| Interesting: | C C | 2 |

Others 1 response: Large and vast world, can't feel about math, frustrated

*Note: As a result of the large number of no responses and varied responses to the third question, it was rephrased for the spring 1992 evaluation.*

## Summary of student responses to questions on understanding, Math 106, Fall 1992

**Responses are given by expected course grade. ( * = no expected grade given)**
**There were 62 respondents.**

1. List three major topics covered in this course.
*Each letter represents 2 responses; letters in parantheses are each 1 response*

| | | |
|---|---|---|
| Derivatives: | A A A A (AB) B B B B B B B B B B (BC) C C C C C C C D D D (F ) | 53 |
| Limits: | A A A (A B) B B B B B B B B (BC) C C C C C C (C D)D (F *) (*) | 45 |
| Functions: | A A (AB) B B B B C C D (*) | 21 |
| Continuity: | (A B) B C C (C D) (F) | 11 |
| Graphing: | A B (B) | 5 |
| No response: | A (B C) (D) | 5 |
| Asymptotes: | B (CD) | 4 |
| Slope: | B (C*) | 4 |
| Polynomials: | (BC) (D*) | 4 |
| Rate of Chg: | A (D) | 3 |
| Tangents: | (AB) (C) | 3 |
| Inequalities: | (AC) | 2 |
| Lines: | C | 2 |
| Algebra: | B | 2 |
| Analy Geom: | (BD) | 2 |

Others 1 response: domain/range, distance, implicit differentiation

2. Describe how this course has changed or not changed what you think mathematics is.

| | | |
|---|---|---|
| Hasn't changed: | A A A A A B B B B B C C C C C C D | 17 |
| No response: | A A A A B B B B B B C C C D * * | 16 |
| Better understanding: | A B C C C C * | 7 |
| Never liked: | B B C C | 4 |
| Appreciate more: | B B B | 3 |
| Not useful: | B B C | 3 |
| Useful: | B B | 2 |
| Difficult: | C D | 2 |

Others 1 response: more respect, hate math, understanding more than memorizing, less stress, extra work, more confident, involved more

3. Describe how this course has changed or not changed your attitude towards mathematics.

| | | |
|---|---|---|
| No response: | A A A A B B B B B C C C C D | 14 |
| More confident: | A A B B B B C C C | 10 |
| Better outlook | A B B B B F | 6 |
| Hasn't changed: | A B B B B C | 6 |
| Hate Math: | A B C C C D | 6 |
| Useful in life: | B C C D | 4 |
| Better understand | B B B | 3 |
| Less confident: | A C * | 3 |
| Less stress: | B C | 2 |
| Appreciate less: | B C | 2 |
| Like math | A * | 2 |

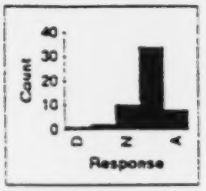
Others 1 response: more stress, still like, less difficult, work harder, depends on teacher, can't wait 'til it's over

## Student Evaluation of Calculus With Review, Part II (Math 166) Spring 1992

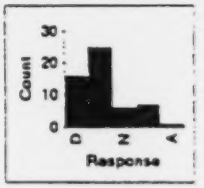
**Number of respondents: 55**

**10 questions on attitudes, 5 questions on use of text materials (histograms of responses below), 3 questions on understanding (responses on page 24). For questions 1–10, D = disagree, N = neutral, and A = agree; for questions 11–15, D = never, N = half the time, and A = always.**

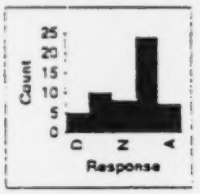
1. This course has improved my ability to solve mathematical problems.



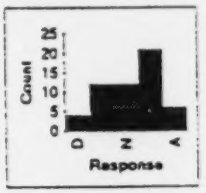

2. I am no better at mathematics than when I began this course.



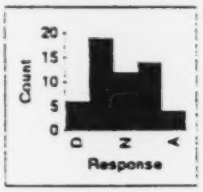

3. Because of this course, I feel able to take further courses in mathematics.



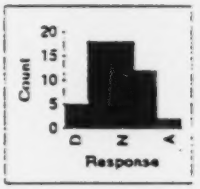

4. Because of this course, I get less anxious about mathematical problems.



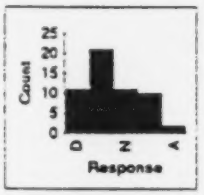

5. Mathematics is just an obstacle to overcome in my college studies.



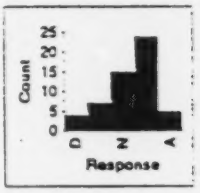

6. Thinking about mathematical problems causes me stress.



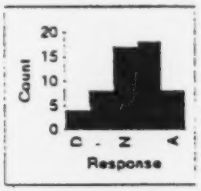

7. Learning mathematics just involves memorizing a set of specific techniques.



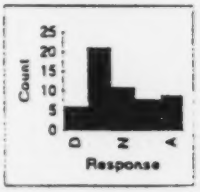

8. The material in this course will be useful to me as I complete my major.



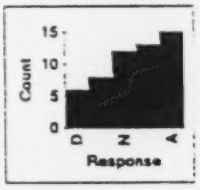

9. Because of this course, I am more aware of the use of mathematics in interpreting the world around me.



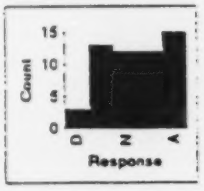

10. I see no need to ever use the material covered in this course again.



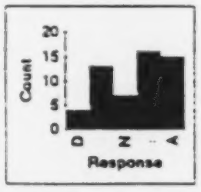

11. Of the assigned readings in this course, I read the handout material.



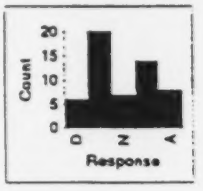

12. Of the assigned readings in this course, I read the Stewart *Calculus* (textbook).



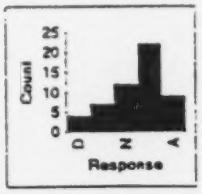

13. I found the handout materials helpful in understanding the course content.




14. I found the Stewart *Calculus* helpful in understanding the course content.




15. I was able to complete my homework assignments.



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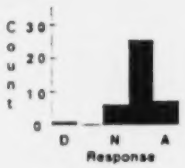
## Student Evaluation of Calculus With Review, Part II (Math 166), Spring 1993

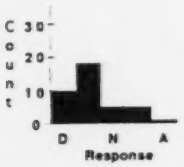
Number of respondents: 39

10 questions on attitudes, 5 questions on use of text materials (histograms of responses below), 3 questions on understanding (responses on page 25). For questions 1–10, D = disagree, N = neutral, and A = agree; for questions 11–15, D = never, N = half the time, and A = always.

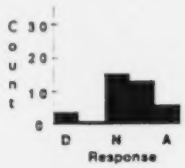
1. This course has improved my ability to solve mathematical problems.



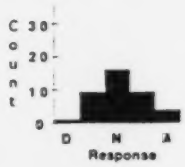

2. I am no better at mathematics than when I began this course.



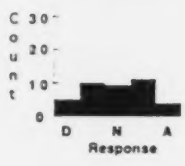

3. Because of this course, I feel able to take further courses in mathematics.



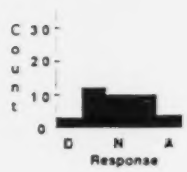

4. Because of this course, I get less anxious about mathematical problems.



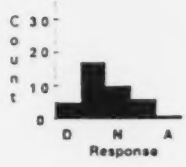

5. Mathematics is just an obstacle to overcome in my college studies.



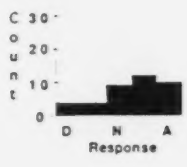

6. Thinking about mathematical problems causes me stress.



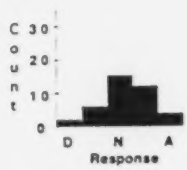

7. Learning mathematics just involves memorizing a set of specific techniques.



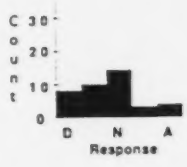

8. The material in this course will be useful to me as I complete my major.



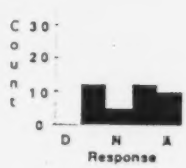

9. Because of this course, I am more aware of the use of mathematics in interpreting the world around me.




10. I see no need to ever use the material covered in this course again.

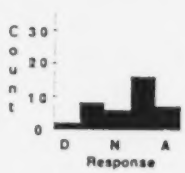



11. Of the assigned readings in this course, I read the handout material

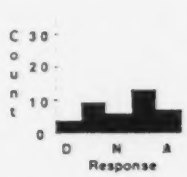



12. Of the assigned readings in this course, I read the Stewart *Calculus* (textbook)

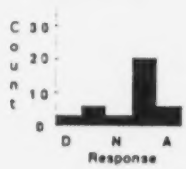
13. I found the handout materials helpful in understanding the course content.




14. I found the Stewart *Calculus* helpful in understanding the course content.




15. I was able to complete my homework assignments.



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## Summary of student responses to questions on understanding, Math 166, Spring 1992

**Responses are given by expected course grade. (* = no expected grade given)**
**There were 55 respondents.**

1. List three major topics covered in the course this semester.

| | | |
|---|---|---|
| Derivative: | AAAAAABBBBBBBBBBBBBBCCCCCCCCCCDDDDD | 36 |
| Integral/antideriv: | AAAAAAABBBBBBBBBBBBCCCCCCD* | 28 |
| Logarithm: | AABBBBBBCCCCCCCCCDD* | 20 |
| Related rates: | ABBBCCCCDDDD | 12 |
| Trigonometry: | AAABBBBCCCC | 11 |
| Min.& Max.: | ABBBBCCD | 8 |
| No response: | AABBCCC | 7 |
| Graphing funct.: | BBBB | 4 |
| Limits: | ABCD | 4 |
| Chain rule: | AB* | 3 |
| Exponential funct.: | BCC | 3 |
| Functions: | AB | 2 |
| Word Problems: | AB | 2 |

Other 1 response: Concavity, Fundamental Theorem of Calculus, Integers, Product & Quotient rules

2. Describe how this course has changed or not changed what you think mathematics is.

| | | |
|---|---|---|
| No response: | AAAAABBBBBBBCCCCCCD* | 20 |
| Useful in real world: | AABBBCCD | 8 |
| Has not changed: | BBCCDD | 6 |
| Better understanding: | BBCCD | 5 |
| Still confused: | BCC | 3 |
| Positive change: | ABB | 3 |
| Math more than numbers: | AC | 2 |
| Not too difficult if do work: | AB | 2 |

Other 1 response: Math defines the perception of the world, see math as a science, basis for all subjects, can visualize problems, still hate math

3. Describe how this course has changed or not changed your attitude towards mathematics.

| | | |
|---|---|---|
| No response: | AAAAABBBBBBBCCCCCCCD | 19 |
| Hasn't changed: | BBBCCDD | 7 |
| Still dislike it: | CCCCC* | 6 |
| More confident: | AABBC | 5 |
| Math useful in life: | ABB | 3 |
| Still like math: | AB | 2 |
| Sees need & it is helpful: | BD | 2 |
| More knowledgeable: | BB | 2 |
| Still difficult: | CC | 2 |

Other 1 response: Not as frustrated, need a good teacher to understand material, open to different math areas, no use outside class, improved attitude, frustrated

## Summary of student responses to questions on understanding, Math 166, Spring 1993

**Responses are given by expected course grade. (* = no expected grade given)**
**There were 39 respondents.**

1. List three major topics covered in the course this semester.

| | | |
|---|---|---|
| Derivative: | A A A A A B B B B B B B B C C C C C C C C C C D D D * | 26 |
| Integral/antideriv: | A A A A B B B B B B B B C C C C C C C D | 20 |
| Trig functions: | A A B B C C C C C C C C C D D D D * | 17 |
| Logarithms: | A A B B B B B B C C C C | 12 |
| Limits: | A C C C D D * | 7 |
| Graphing funct.: | A B B B C C | 6 |
| Exponential funct.: | A A A | 3 |
| Inverse functions: | A B C | 3 |
| Summations: | A B | 2 |
| Related rates: | A D | 2 |

Other 1 response: Functions, vertical/horizontal asymptotes, one-to-one

2. Describe how this course has changed or not changed what you think mathematics is.

| | | |
|---|---|---|
| No response: | A B B B C C D * | 8 |
| Useful in real world: | A A A B B C C C | 8 |
| Has not changed: | A B B C C D | 6 |
| Math is hard: | C C C D D D | 6 |
| Better understanding: | A B D | 3 |
| Enjoy math: | B B C | 3 |
| Waste of time: | C C | 2 |

Other 1 response: Less imtimidated, hate math, math is easy

3. Describe how this course has changed or not changed your attitude towards mathematics.

| | | |
|---|---|---|
| Hasn't changed: | A B B B C C C D | 8 |
| No response: | A C C C C D | 6 |
| Like math: | A B B C D | 6 |
| Attitude improved: | A B B C D | 5 |
| Still dislike it: | A C C D | 4 |
| Math is boring: | B C | 2 |
| Causes stress | A C | 2 |
| Math is applicable: | A B | 2 |

Other 1 response: Difficult, challenging

COURSE EVALUATION FOR MATH 106_____
section number

CHARACTERISTICS OF THE STUDENT

Please circle your gender: male female

Please enter the grade you expect to earn in this course _____

EVALUATION OF THE COURSE

Please respond to each of the following statements about this course by checking the appropriate box:

| | STRONGLY DISAGREE | DISAGREE | NEITHER AGREE NOR DISAGREE | AGREE | STRONGLY AGREE |
|---|---|---|---|---|---|
| 1. This course has improved my ability to solve mathematical problems. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 2. I am no better at mathematics than when I began this course. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 3. Because of this course, I feel able to take further courses in mathematics. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 4. Because of this course, I get less anxious about mathematical problems. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 5. Mathematics is just an obstacle to overcome in my college studies. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 6. Thinking about mathematical problems causes me stress. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 7. Learning mathematics just involves memorizing a set of specific techniques. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 8. The material in this course will be useful to me as I complete my major. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 9. Because of this course, I am more aware of the use of mathematics in interpreting the world around me. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 10. I see no need to ever use the material covered in this course again. | ( ) | ( ) | ( ) | ( ) | ( ) |

Please respond to each of the following statements about the course materials by checking the appropriate box:

| | ALMOST NEVER | SOMETIME | HALF OF THE TIME | USUALLY | ALMOST ALWAYS |
|---|---|---|---|---|---|
| 1. Of the assigned readings in this course, I read: | | | | | |
| a. the handout material | ( ) | ( ) | ( ) | ( ) | ( ) |
| b. the Stewart *Calculus* (textbook) | ( ) | ( ) | ( ) | ( ) | ( ) |
| 2. I found the handout materials helpful in understanding the course content. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 3. I found the Stewart *Calculus* (textbook) helpful in understanding the course content. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 4. I was able to complete my homework assignments. | ( ) | ( ) | ( ) | ( ) | ( ) |

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please answer the questions on the reverse side

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1. List three major topics covered in this course.

2. Describe how this course has changed or not changed what you think mathematics is.

3. Describe how this course has changed or not changed how you <u>feel</u> about mathematics.

# Appendix 3

## Announcements and program materials from the dissemination conference held at Moravian College, June 18-19, 1993

# Conference on Integration of Precalculus with Calculus
# Moravian College, June 18-19, 1993

This conference will focus on several issues that surround the development of a calculus course for underprepared students that integrates precalculus concepts as they are needed in a first course in calculus. It caps a two-year FIPSE-funded project in which faculty at Moravian College and Northampton Community College developed and class-tested materials especially designed for this course. The materials are to be used along with a standard calculus text.

The conference will feature an overview of precalculus and calculus reform nationwide, given by Susan Foreman (from the Mathematical Sciences Education Board), an open discussion by faculty from four institutions that have "integrated" courses, contributed papers, and a display of supporting materials.

Contributed papers are invited on the following topics:

- Courses that integrate precalculus with calculus
- Research in learning precalculus-calculus
- Teaching calculus to the underprepared student or nontraditional student
- Assessment of attitudes and mathematical understanding
- The role of technology in a precalculus-calculus course.

Abstracts must be submitted for review by April 15, 1993.
The registration deadline for the conference is May 30, 1993.

For further information, contact:

Doris Schattschneider
Department of Mathematics
Moravian College
1200 Main Street
Bethlehem, PA 18018-6650.

email: schattdo@moravian.edu
Telephone: 215 861 1373
FAX: 215 861 3919

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INFORMATION: The Onsager Symposium, The Norwegian Institute of Technology, N-7034 Trondheim, Norway; email: onsager93@imf.unit.no.

2–5. **Ninth Biennial Conference of the Association of Christians in the Mathematical Sciences**, Westmont College, Santa Barbara, CA. (Sep. 1992, p. 773)

6–9. **Annual Meeting of the Statistical Society of Canada**, Wolfville, Nova Scotia, Canada. (Feb. 1992, p. 149)

6–12. **Analysis auf Kompakten Varietäten**, Oberwolfach, Federal Republic of Germany. (Jan. 1992, p. 56)

7–10. **SIAM Conference on Mathematical and Numerical Aspects of Wave Propagation Phenomena**, University of Delaware, Newark, DE. (Jul./Aug. 1992, p. 631)

7–10. **The Eighth Haifa Matrix Theory Conference**, Technion, Haifa, Israel. (Nov. 1992, p. 1119)

7–11. **IMA Tutorial: Mathematical Theory which Has become an Integral Part of Modern Financial Economics**, Institute for Mathematics and its Applications, University of Minnesota, Minneapolis, MN. (Nov. 1991, p. 1172)

7–11. **Colloque International en l'Honneur de G. Freiman. La Methode Additive Inverse et ses Applications**, CIRM, Marseille, France. (Nov. 1992, p. 1119)

* 7–11. **Art and Mathematics Conference (AM93)**, State University of New York, Albany, NY.

PROGRAM: AM93 is an international interdisciplinary conference relating art and mathematics. The emphasis is on visualization with examples from architecture, geometry, graphics, quilts, painting, sculpture, and topology.
INVITED SPEAKERS: T. Banchoff, J. Conway, P. Davis, S. Dickson, M. Emmer, H. Ferguson, Z. Hecker, C. Meadmore, T. Milkowski, C. Perry, R. R. Shearer, and K. Snelson. There will be panel discussions, a slide and video registry, and space available for displays.
INFORMATION: N. Friedman, Dept. of Math., SUNY-Albany, Albany, NY 12222; FAX: 518-442-4731; email: artmath@math.albany.edu; tel: 518-442-4621.

7–12. **International Conference in Honour of Bernard Malgrange**, Grenoble, France. (Nov. 1992, p. 1119)

7–13. **Workshop on Pattern Formation and Cellular Automata**, Fields Institute for Research in Mathematical Sciences, Waterloo, Ontario. (Apr. 1992, p. 352)

* 13–18. **The Householder Symposium XII Meeting on Numerical Algebra**, UCLA Conference Center, Lake Arrowhead, CA.

PROGRAM: This meeting is the twelfth in a series, previously called the Gatlinburg Symposia, renamed to honor Alston S. Householder. The meeting will be very informal. Extended talks are given during the day and special workshops organized by the participants in the evening. The following topics are emphasized: parallel computation issues, signal and image processing, control, wavelets, nonsymmetric conjugate gradient methods, domain decomposition and multilevel methods and industrial problems.
INFORMATION: Householder 93, c/o B. Dalton, Dept. of Math., Univ. of California, Los Angeles, 405 Hilgard Ave., Los Angeles, CA 90024-1555; householder93@math.ucla.edu.

13–19. **Differential-Algebraic Equations: Theory and Applications in Technical Simulation**, Oberwolfach, Federal Republic of Germany. (Jan. 1992, p. 56)

14–17. **The Fifth Asian Logic Conference**, National University of Singapore, Singapore. (May/Jun. 1992, p. 496)

14–18. **IMA Workshop on Mathematical Finance**, Institute for Mathematics and its Applications, University of Minnesota, Minneapolis, MN. (Nov. 1991, p. 1172)

14–18. **Linear Logic Workshop**, Mathematical Sciences Institute, Cornell University, Ithaca, NY. (May/Jun. 1992, p. 496)

14–18. **Homologie des Algebres et Applications**, CIRM, Marseille, France. (Nov. 1992, p. 1119)

15–17. **IEEE Computer Society Conference on Computer Vision and Pattern Recognition**, Omni Park Central, New York City, NY. (Nov. 1992, p. 1119)

15–18. **Third IMACS International Workshop on Qualitative Reasoning and Decision Technologies–QR&DT-3**, Polytechnique of Barcelona, Spain. (Jan. 1992, p. 56)

* 17–19. **ATLAST 1993 Linear Algebra Workshops**, Michigan State University, East Lansing, MI.

PROGRAM: ATLAST is an NSF-ILAS Project to Augment the Teaching of Linear Algebra through the use of Software Tools. The project will offer five faculty workshops during the summer of 1993 on the use of software in teaching linear algebra. Workshop participants will learn about existing commercial linear algebra software packages and will be trained in the use of the MATLAB software package. Attendees will learn how to effectively incorporate excercises and laboratories into undergraduate linear algebra courses. Participants will learn to design computing exercises at a level suitable for assigning to an undergraduate linear algebra class. These exercises will be class-tested during the school year following the workshop and then submitted to the project director for inclusion in an edited manual which will be distributed to the attendees. The ATLAST Project provides room and board for participants attending the workshops. Participants will also receive a stipend of $200 for their submitted exercises.
DIRECTORS: S.J. Leon, Director (ILAS Education Committee, Univ. of Massachusetts, Dartmouth) and Asst. Director R.E. Faulkenberry (Univ. of Massachusetts, Dartmouth).
WORKSHOP PRESENTER: S.J. Leon, UMass Dartmouth.
APPLICATIONS: All teachers of undergraduate linear algebra courses at colleges or universities in the U.S. are invited to apply. Deadline for applications is March 12, 1993. Each workshop will be limited to thirty participants.
INFORMATION: R. Faulkenberry, ATLAST Project Assistant Director, Dept. of Math., Univ. of Massachusetts Dartmouth, North Dartmouth, MA 02747; 508-999-8928; FAX: 508-999-8901; email: atlast@umassd.edu.

* 18–19. **Conference on Integration of Precalculus with Calculus**, Moravian College, Bethlehem, PA.

PROGRAM: The program will focus on issues related to teaching calculus to underprepared students, and special courses that integrate precalculus concepts as they are needed in a first course in calculus.
CONFERENCE TOPICS: Courses that integrate precalculus with calculus, research in learning precalculus-calculus, teaching calculus to the underprepared student or to the nontraditional student, assessment of attitudes and mathematical understanding, the role of technology in a precalculus course.
CALL FOR PAPERS: Abstracts (1 page or less) must be submitted for review (address below) by April 1, 1993.
INFORMATION: Registration deadline is May 15, 1993. For further information, write to: D. Schattschneider, Dept. of Math., 1200 Main St., Moravian College, Bethlehem, PA 18018-6650; 215-861-1373; FAX: 215-861-3919; email: schattdo@moravian.edu.

20–23. **Eighth Annual IEEE Symposium on Logic in Computer Science (LICS)**, Montreal, Canada. (Nov. 1992, p. 1119)

20–26. **Konvexgeometrie**, Oberwolfach, Federal Republic of Germany. (Jan. 1992, p. 56)

21–25. **Twenty-second Conference on Stochastic Processes and their Applications**, Amsterdam, The Netherlands. (Sep. 1992, p. 773)

21–25. **Graphs on Surfaces**, Johns Hopkins University, Baltimore, MD. (Sep. 1992, p. 773)

21–25. **Fifth International Conference on**



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# Faculty Development Workshops

## For Mathematicians who Teach Statistics

If you teach undergraduate statistics courses, but are not yourself a statistician, you are invited to participate in one of a series of regional week-long workshops sponsored by MAA, with anticipated support from the National Science Foundation's Undergraduate Faculty Enhancement Program. These workshops are designed to help teachers at two- and four-year colleges implement the recommendations of the MAA's Focus Group on Statistics, as reported in *UME Trends* (October, 1991), and more fully in *Heeding the Call for Change* (MAA Notes No. 22, Lynn Steen, ed.).

Both the practice of statistics and its intellectual framework have been changing rapidly over the last two decades, largely in response to the computer revolution. To address these changes, the workshops will feature four days of presentations by leading applied statisticians, whose lectures will be linked to chapters in *Perspectives in Contemporary Statistics* (MAA Notes No. 21, David Hoaglin and David Moore, eds.). For example, two days of a workshop might be devoted to the practice of exploratory data analysis and to the corresponding conceptual shift which places greater emphasis on the "model-data dialog" at the expense of classical hypothesis testing. Another two days might be spent on the comparatively new area of regression diagnostics, which is reshaping the way statisticians choose and evaluate models based on fitting lines and curves to data. Yet another area of rapid change is statistical process control, which has grown well beyond its origins in industry to offer, what some regard as, a new theoretical frame for all of applied statistics.

Parallel with changes in the field of statistics, recent research on how students learn has documented the importance of increasing the opportunities for active, hands-on learning by students of statistics. In this spirit, the workshops will emphasize topics and activities that lend themselves to direct use with students. Faculty who come to the workshops will be given opportunities (indeed expected!) to participate actively in their own learning. In particular, faculty will have the chance to try out modules developed by Richard Scheaffer's NSF-funded Activity-Based Statistics (ABS) Project.

Each workshop will bring together 24 faculty who, though they have neither recent professional training nor a graduate degree in statistics, have been called on by their departments to teach statistics courses. They would also like to learn more about the subject and the resources available to those who teach it. Prior to each workshop, the 24 participants, the workshop coordinator, and the statistician presenters will be linked by e-mail so they can introduce themselves and confer with each other about their backgrounds, the courses they teach, and their goals for the workshop. This is to ensure a good match between the needs and interests of the participants, the presentations, and the activities at the workshop. Participants need not have prior experience with e-mail; means of access and help getting started will be provided as needed.

Four days of the week-long workshop will be divided between presentations by statisticians and related computer lab activities. The rest of the time teams of four participants, with guidance and assistance from the workshop coordinator, will each design and carry out projects involving collection and analysis of statistical data or activities suitable for a statistics lab. After the workshop ends, team members will remain networked with one another, with the statistician presenters, and with the workshop coordinator, first during the rest of the summer as they develop and adapt their projects for use in their own teaching, and then throughout the fall as they try out their projects in the classroom. Each regional workshop will hold a one-day reunion after the tryout period, in connection with a regional or national meeting of the MAA.

Participants or their home institutions are expected to cover the cost of travel to the workshop site. Each workshop will provide room and board for 24 participants. In addition, participants will receive course materials which will include a copy of *Perspectives in Contemporary Statistics*, a copy of the statistical analysis package used at the workshop, and $100 towards the cost of travel, room, and board for the one-day reunion.

The workshops originate from the Committee on Undergraduate Statistics appointed jointly by the MAA and The American Statistical Association (ASA). The project steering committee includes Donald Bushaw and Ann Watkins from the MAA, and David Moore and Richard Scheaffer from the ASA.

Project Directors George Cobb of Mount Holyoke College and Mary Parker of Austin Community College are sending more detailed information on the program and application procedures to department chairs. The application deadline for the 1993 workshops is 2 April 1993. For further information, please contact April White, STATS Project Registrar, The Mathematical Association of America, 1529 Eighteenth Street NW, Washington DC 20036-1385 (Phone: 202-387-5200, FAX: 202-265-2384).

### 1993 Statistical Thinking and Teaching Statistics (STATS) Workshops

**6-13 June 1993 University of Iowa, Iowa City, IA**

**Jonathon D. Cryer, Coordinator**

**13-20 June 1993 Bowdoin College, Brunswick, ME**

**Rosemary A. Roberts, Coordinator**

## Conference on Integration of Precalculus with Calculus

## Moravian College, 18-19 June 1993

Moravian College in Bethlehem, Pennsylvania will host a conference on 18-19 June 1993 focusing on several issues that surround the development of a calculus course for under-prepared students. This will cap a two-year FIPSE-funded project in which faculty at Moravian College and Northampton Community College developed and class-tested materials especially designed for this course, to be used along with a standard calculus text.

Contributed papers are invited on the following topics:

Courses that integrate precalculus with calculus; Research in learning precalculus-calculus; Teaching calculus to the under-prepared student or to the non-traditional student; Assessment of attitudes and mathematical understanding; The role of technology in a precalculus-calculus course. Abstracts must be submitted for review by 1 April 1993. The registration deadline for the conference is 15 May 1993.

For further information on the conference, including registration materials, or to submit an abstract, write to: Doris Schattschneider, Department of Mathematics, 1200 Main Street, Moravian College, Bethlehem, PA 18018-6650. E-mail: Schattdo@moravian.edu.

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(MSRI) in Berkeley, California, for undergraduate and graduate student women. The program is specifically designed to invite talented young women to consider a career in pure mathematics in general and in the area of algebraic geometry in particular. Women students admitted to the Summer Geometry Institute in Park City will also be invited to attend a special two-week program at MSRI in May 1993. There they will be offered a program of an introduction to algebraic geometry and participation in small working groups with individual mentoring, organized especially for them by participants of the Special Year in Algebraic Geometry at MSRI. Mentors from this program will then accompany the group to the Summer Institute in Park City in June and July to continue their support activities.

For more information about the women's joint program with MSRI, please write to either Professor Karen Uhlenbeck (Department of Mathematics, University of Texas, Austin, TX 78712; email: uhlen@math.utexas.edu) or Professor Robert Bryant (Department of Mathematics, Duke University, Durham NC 27706; bryant@math.duke.edu). Please include your email address, mailing address, and daytime phone number.

For general information about the Summer Geometry Institute and application forms, please contact the Regional Geometry Institute, 18C de Trobriand Street, Fort Douglas, Salt Lake City, UT 84113. Phone: (801) 585-3488. Fax: (801) 585-5793. Email: rgi@math.utah.edu

The Summer Geometry Institute specifically invites applications from women and members of minority groups.

### NFS-CBMS Regional Research Conferences in the Mathematical Sciences

Contingent upon NSF funding, five NSF-CBMS regional research conferences will be held between May and August of 1993. These conferences are intended to stimulate interest and activity in mathematical research. Each five-day conference features a distinguished lecturer who delivers ten lectures on a topic of important current research in one sharply focussed area of the mathematical sciences. The lecturer subsequently prepares an expository monograph based upon these lectures, which is normally published as a part of a regional conference series.

Pending funding, support for about 30 participants is provided. The conference organizer invites both established researchers and interested newcomers, including postdoctoral fellows and graduate students, to attend.

The five anticipated conferences are: "Semiparametric Mixture Models: Theory and Methods, with Applications in Empirical Bayes, Measurement Error, and Random Effects Models," Bruce G. Lindsay, lecturer, mid-May, University of South Carolina at Columbia, John M. Grego, organizer, 803-777-4651, n540006@univscvm; "Applications of the Representation Theory of Quantum Affine Lie Algebras to Solvable Lattice Models," Tetsuji Miwa, lecturer, June 1–5, North Carolina State University, Kailash C. Misra, organizer, 919-515-3320, misra@ncsuvm.cc.ncsu.edu; "Compensated Compactness, Homogenization and H-Measures," Luc C. Tartar, lecturer, June 28 – July 3, University of California at Santa Cruz, Maria E. Schonbek, organizer, 408-459-4657; "Equivariant Homotopy and Cohomology," J. Peter May, lecturer, August 2–6, University of Alaska at Fairbanks, Robert J. Piacenza, organizer, ffrjp@alaska.bitnet, 907-474-7772; and "Classification of Amenable Subfactors and Related Topics," Sorin T. Popa, lecturer, August 24–28, University of Oregon, N. Christopher Phillips, phillips@bright.uoregon.edu, 503-346-4714.

Information about an individual conference may be obtained by contacting the conference organizer. Information about the series may be obtained by writing CBMS, 1529 Eighteenth St., NW, Washington, DC 20036; 202-293-1170.

Proposals for the 1994 NSF-CBMS Regional Research Conferences in the Mathematical Sciences are requested. The closing date for applications is April 1, 1993. For a brochure giving the program description, review criteria, and information on format and submission of proposals, call or write CBMS as above.

### Conference on Integration of Precalculus with Calculus, Moravian College, June 18-19, 1993

Access to calculus for students who do not have adequate preparation is an issue that every college and university has to face. The standard approach of requiring a precalculus course (or a tier of various remedial courses) for such students prior to enrolling in the first calculus course is one that has limited success. Often the majority of these students never make it beyond the remedial course. Recently several institutions have tried a new approach: a

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slower-paced one-year course that covers the content of the usual first semester course in calculus while providing the necessary background and review of precalculus material as it is needed.

Moravian College in Bethlehem, Pennsylvania will host a conference on June 18–19, 1993 that will focus on several issues that surround the development of a calculus course for underprepared students. This will cap a two-year FIPSE-funded project in which faculty at Moravian College and Northampton Community College developed and class-tested materials especially designed for this course, to be used along with a standard calculus text.

The conference will feature an overview by Susan Foreman (of the Mathematical Sciences Education Board) of different initiatives by various institutions to make calculus accessible to a wide audience, an open discussion by faculty at several institutions who have developed "integrated" courses at their institutions, contributed papers, and a display of materials.

Contributed papers are invited on the following topics: courses that integrate precalculus with calculus, research in learning precalculus-calculus, teaching calculus to the underprepared student or to the nontraditional student, assessment of attitudes and mathematical understanding, and the role of technology in a precalculus-calculus course. Abstracts must be submitted for review by April 1, 1993. The registration deadline for the conference is May 15, 1993.

For further information on the conference, including registration materials, or to submit an abstract, write to: Doris Schattschneider, Department of Mathematics, 1200 Main Street, Moravian College, Bethlehem, PA 18018-6550; email: schattdo@moravian.edu.

### Joint Summer Research Conferences in the Mathematical Sciences

The 1993 Joint Summer Research Conferences in the Mathematical Sciences will be held at the University of Washington, Seattle, from July 10 to August 6. It is anticipated that the series of conferences will be supported by grants from the National Science Foundation and other agencies.

The conferences are: "Curvature equations in conformal geometry," July 10–16; "Multivariable operator theory," July 10–18; "Spectral geometry," July 17–23; "Recent developments in the inverse Galois problem," July 17-23; "Mathematics of superconductivity," July 24–30; "Distributions with fixed marginals, doubly stochastic measures, and Markov operators," July 31 – August 6; and "Applications of hypergroups and related measure algebras," July 31 – August 6.

The deadline for receipt of requests for information is March 1, 1993. For detailed descriptions of the topics of the conferences, see the November 1992 AMS *Notices*.

## WOMEN IN MATHEMATICS, 1991-92

| Department | Tenured | | Untenured | | Tenure Track/ Could Lead to Tenure | |
|---|---|---|---|---|---|---|
| | Total | Female | Total | Female | Total | Female |
| UC-Berkeley | 60 | 2* | 12 | 3 | 2 | 0 |
| Caltech | 13 | 0 | 6 | 0 | 1 | 0 |
| Chicago | 25 | 0 | 24 | 2 | 6 | 0 |
| Columbia | 14 | 1** | 12 | 0 | 0 | 0 |
| Harvard | 17 | 1 | 14 | 3 | 1 | 0 |
| MIT | 40 | 0 | 38 | 4 | 12 | 1 |
| Michigan | 49 | 1 | 38 | 6 | 3 | 1 |
| Princeton | 31 | 0 | 28 | 7 | 22 | 5 |
| Stanford | 23 | 0 | 9 | 1 | 2 | 0 |
| Yale | 16 | 0 | 11 | 1 | 3 | 0 |
| Total | 288 | 5 | 192 | 27 | 52 | 7 |

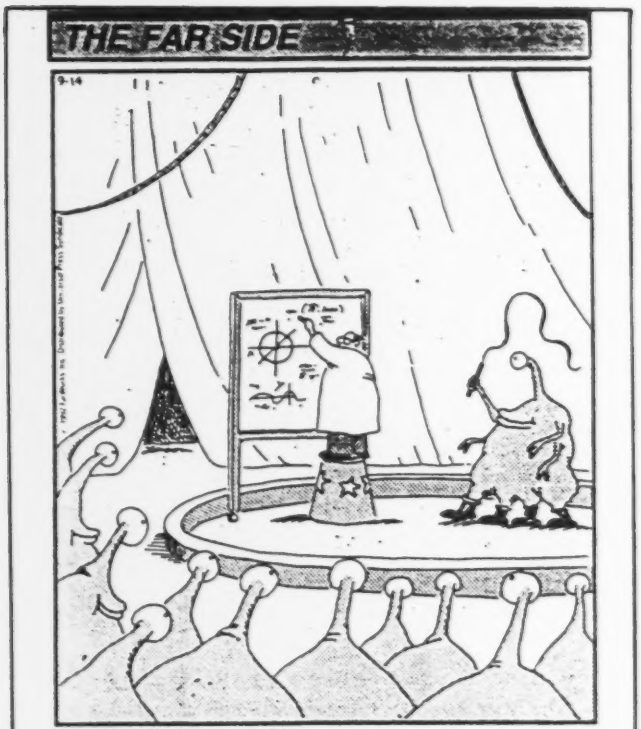
* One has a joint appointment with UCLA.
** Tenured at Barnard

**Women in math update.** Last year, *Science* published a table on the number of women who have tenure or are in tenure-track positions at 10 math departments in the United States. That table caused considerable controversy, partly because of some confusion over its categories. In the interest of accuracy, we are publishing a carefully revised version of the table, updated to the 1991-1992 academic year. "Untenured" is here used to mean all full-time members of a department who do not have tenure, including both tenure-track and non-tenure-track positions. "Tenure-track," a subset of the untenured group, means members of a department with appointments at the end of which the member must automatically be considered for tenure. Columbia, Harvard, and Yale report no tenure-track appointments in this strict sense. Yale notes that "assistant professors have been promoted to tenured positions

Conference on

# Integration of Precalculus with Calculus

Moravian College
June 18-19, 1993




Abducted by an alien circus company, Professor Doyle is forced to write calculus equations in center ring.

Support for this conference has been provided by
The Lehigh Valley Association of Independent Colleges
The Fund for the Improvement of Post-Secondary Education,
U. S. Department of Education

Special contributions:
W. H. Freeman & Company, Publisher (Continental Breakfast)
Addison Wesley Publishing Company (Closing Reception)



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## Conference on Integration of Precalculus with Calculus
Moravian College, June 18-19, 1993

### Schedule of Events

**Friday, June 18 Church Street campus**

5:30—6:30 p.m. Registration and get-acquainted reception, Payne Art Gallery

6:30 — 7:45 p.m. Dinner, Clewell Hall dining room

8:00 p.m. Talk, "The Art of Mathematics," Professor Jerry King, Lehigh University. (King's recent book of the same title has been the subject of interesting reviews and talk show discussion.) Peter Hall.

9:00 p.m. Dessert reception, Hearst Hall.

**Saturday, June 19 Main campus:** Haupert Student Union and Collier Hall of Science

8:15–9:15 a.m. Registration and Continental breakfast, Lounge, Haupert Student Union

9:15 a.m. Greetings, Martha Reid, Dean of the College, Prosser Auditorium

9:30–10:30 a.m., Prosser Auditorium
Keynote address by Dr. Susan Forman, Mathematical Sciences Education Board.

10:30–11:00 a.m. Coffee break, Lounge

11:00 a.m.–12:15 p.m., Prosser Auditorium
Panel and open discussion. Panelists from institutions that have courses that integrate precalculus with the first course in calculus will discuss these courses. Dennis Ebersole (Northampton Community College and Moravian College), Nancy Baxter (Dickinson College), Michael Rogers (Amherst College), Lenore Frank (SUNY at Stonybrook).

12:30 p.m. Lunch, Main Dining Hall, Haupert Student Union

2:00–4:15 p.m. Dana Lecture Hall and Mellon Lecture Hall, Collier Hall of Science
Contributed papers. Two Parallel Sessions (see schedule of talks on next page).

4:15 p.m. Closing Reception, Lounge, Haupert Student Union.

NOTE: Throughout the day, there will be opportunity for participants to view materials designed for use in various integrated precalculus–calculus courses, as well as obtain information available about such courses currently in place. Materials will be on display in the main lounge of the Haupert Union Building.

# Conference on the Integration of Precalculus with Calculus

**Schedule of Contributed Talks**
**2:00 p.m.— 4:15 p.m. Saturday, June 19, 1993**
**Collier Hall of Science**

There will be two parallel sessions. Participants may move between the sessions, which are in adjacent rooms. Abstracts of the presentations are on the next three pages.

| | Session A, Dana Lecture Hall | Session B, Mellon Lecture Hall |
|---|---|---|
| 2:00–2:20 | N. Baxter, C. Fratto & L. Mellot<br>Dickinson College<br>"Integrating Precalculus with Calculus: The Workshop Approach" | E. J. Manfred<br>U.S. Coast Guard Academy<br>"The Precalculus Program at the U.S. Coast Guard Academy" |
| 2:30–2:45 | J. A. Seebach<br>St. Olaf College<br>"Words, Pictures, and Symbols: Interlacing Precalculus with Calculus" | I. Peterburgsky<br>Suffolk University<br>"From Fear and Hostility to Interest, Understanding and Enthusiasm" |
| 2:50–3:05 | P. Bolstad<br>St. Olaf College<br>"Modifications of a Precalculus/Calculus course to meet special needs" | J. Hefferon<br>St. Michael's College<br>"Some Problems Integrating Precalculus with Calculus" |
| 3:10–3:25 | K. Somers & A. Sevilla<br>Moravian College<br>"Calculus with Review: Course materials and evaluation" | L. Emerson<br>Western New England College<br>"Concurrent Precalculus/Calculus at Western New England College" |
| 3:30–3:50 | Joyce Williams<br>Univ. of Massachusetts at Lowell<br>"Integrated course at University of Massachusetts at Lowell | L. T. Kulich<br>Illinois Benedictine College<br>"A Precalculus/Calculus course at Illinois Benedictine College" |
| 3:55–4:15 | A. Azzolino<br>Mathematical Concepts, Inc.<br>"Seeing and Touching in Mathematics" | D. A. Cohen<br>Union County College<br>"Discovery Textbook with Extensive Review" |
| 4:15 | Closing Reception, Haupert Union Lounge | |

## ABSTRACTS OF CONTRIBUTED TALKS

2:00 - 2:20 Dana Lecture Hall

"Integrating Precalculus With Calculus: The Workshop Approach"

*Nancy Baxter, Christa Fratto '94 and Linda Mellott '94, Dickinson College*

Workshop Calculus is a two course sequence for students who are not prepared to enter our regular calculus sequence, but who need calculus for further study in social sciences, natural sciences or mathematics. Workshop Calculus integrates precalculus with calculus. It is designed to prepare students to enter Calculus II, thereby providing an alternate entry point into the field of mathematics. Hopefully, it will inspire students to continue their study of mathematics.

This presentation will give an overview of the Workshop Calculus Project, which is supported by grants from the Fund for Improvement of Post Secondary Education (FIPSE) and the National Science Foundation. Specific examples of integrating precalculus with calculus will be given and the role of technology will be emphasized. Design and implementation issues will be discussed. Sample materials will be available upon request.

2:00 - 2:20 Mellon Lecture Hall

"The Precalculus Programs at the United States Coast Guard Academy"

*Ernest J. Manfred, U. S. Coast Guard Academy*

The United States Coast Guard Academy is one of five federal service academies. The core curriculum requires a minimum of two calculus courses plus a calculus-based probability and statistics course. Since 1979, the Mathematics Department has used a multiple linear regression model that predicts grades in the first semester calculus course. Two independent variables in the model are algebra and trigonometry diagnostic exams. An item analysis has been done each year to identify those topics that present the most difficulty. This paper will discuss the topics and two approaches to the precalculus program.

2:30 - 2:45 Dana Lecture Hall

"Words, Pictures and Symbols: Interlacing Precalculus w/ Calculus"

*J. Arthur Seebach, Jr., St. Olaf College*

Our precalculus students were, with good reason, unmotivated and antagonistic due to their previous "mathematical" experiences. We chose a literate calculus book *Calculus from Graphical, Numerical, and Symbolic Points of View* by Ostebee and Zorn, and started into new material with lots of pictures and descriptions not heavily dependent on algebraic skills. With "how things change" as our theme we took a scenic route through the derivative before regrouping to develop technical skills in which they now showed genuine interest. In the second semester, after completing a tour of derivatives and their associated antiderivatives, we tackled interesting word problems, including several types of differential equations, emphasizing "translation" of graphical and symbolic forms. Finally we spent 5 weeks on integration with the theme "Area is a paradigm for product!"

2:30 - 2:45 Mellon Lecture Hall

"From Fear and Hostility to Interest, Understanding, and Enthusiasm"

*Dr. Irina Peterburgsky, Suffolk University*

The paper is devoted to our experience in teaching precalculus and calculus to biology freshmen at Suffolk University (academic year 1992-1993). The attempts to teach mathematics to biology students at Suffolk using our traditional teaching schemes and class setting remained unsuccessful for a number of years. A modified experimental teaching approach was developed and implemented at the Department of Mathematics and Computer Science during the last academic year. We describe and analyze our approaches, observations and results.

2:50 - 3:05 Dana Lecture Hall

"Modifying Precalculus/Calculus for Special Needs Students"

*Peder A. Bolstad, St. Olaf College*

One section of the new Calculus with Algebra course at St. Olaf College last fall was supported by a federal Student Support Services grant. The target population for SSS grants is low income and first generation college students. In my two most recent (89-90 and 90-91) precalculus classes, 35% of the general population and only 10% of the SSS students received grades of B- or better. The purpose of this special section was to try to improve performance by reducing class size and offering additional daily problem sessions with an upperclass mathematics tutor. More than half of the students in this SSS section received grades of B- or better and most of those went on to another math course in the spring semester. These results are very encouraging both for the new course and for special needs classes, and some improvements can be made for next year.

2:50 - 3:05 Mellon Lecture Hall

"Some Problems Integrating Precalculus with Calculus"

*Jim Hefferon, Saint Michael's College*

St. Michael's experimented with giving our Precalculus refresher in a Calculus context. The audience was students in fields like Biology and Economics. We were trying to draw them into the main Calculus sequence.

We had some success: 30% of the class went to Calculus II, compared to none before this course.

I will discuss some problems we had, that a school thinking about offering this course should consider. No suitable text was found. In particular, the books we saw had few substantive and genuine applications in the audience's areas of interest. Equally troubling was that students who did not succeed did quite badly.

3:10 - 3:25 Dana Lecture Hall

"Calculus with Review: Course Materials and Evaluation"

*Alicia Sevilla and Kay Somers, Moravian College*

In 1988 the Mathematics Department at Moravian College introduced a two-semester course that integrates precalculus topics *as needed* into a first course in calculus. As a result, a separate precalculus course is no longer taught at Moravian. The talk will include a brief description of the integrated course, a description of the course materials that have been developed, and a discussion of some outcomes. The course materials were developed with funding from a FIPSE grant and are designed for use with a standard calculus text.

3:10 - 3:25 Mellon Lecture Hall

"Concurrent Precalculus/Calculus"

*Lloyd Emerson, Western New England College*

The history and structure of a two-semester sequence of massively integrated Precalculus/Calculus courses taught at WNEC for the past four years will be described briefly. The potential advantages of a second exposure during the second semester to such topics as the use of the chain rule, differentiation using the implicit method, and the solution of problems involving related rates will be argued. Finally, some recent teaching experience involving the use of an upperclass TA to enhance classroom interaction during highly structured small group problem-solving activities will be reported.

3:30 - 3:45 Dana Lecture Hall

"Integrated Precalculus and Calculus Using Standard Texts"

*Joyce Williams, presenter, Guntram Mueller, and Marvin Stick,*
*University of Massachusetts Lowell*

College students who need preparatory work in algebra and trigonometry usually do not want to be placed in a mathematics course they regard as remedial. To address this problem in motivation, we have integrated needed algebra and trigonometry with first semester calculus topics and have spread the material over two semesters. Following these two semesters, students are placed into the second semester of the standard calculus sequence. To ease the transition, the same calculus book is used for both tracks. A separate standard book for the precalculus topics is used. Studies show that those originally thought to be at risk are performing at a level comparable to the level of the standard group.

3:30 - 3:50 Mellon Lecture Hall

"A Precalculus/Calculus Course at IBC"

*Lisa Townsley Kulich, Illinois Benedictine College*

In response to student difficulty in transition from Precalculus to Calculus I, Illinois Benedictine College eliminated Precalculus from the fall '91 schedule, and replaced it with Introduction to Calculus I. The 2-semester, 10-credit sequence integrates Precalculus and Calculus I topics, chiefly by including the concepts of precalculus mathematics as motivated by the calculus concepts. Students who lack the necessary skills to enter a traditional calculus course are introduced to calculus at a slower pace, incorporating the review of precalculus skills as they become necessary. The sequence incorporates a strong emphasis on functional concepts and graphs, and includes a laboratory component using DERIVE software and emphasizing skill in communicating mathematics. The student who satisfactorily completes both terms will be prepared to enter Calculus II. The speaker (and creator of the course) presents an overview of the course.

3:55 - 4:15 Dana Lecture Hall

"Seeing and Touching in Mathematics"

*Agnes Azzolino, Mathematical Concepts, Inc.*

Move from a lecture, to demonstration, or even to a workshop format.

Model the expansion of a binomial. Consider the ambiguous case of the Sine Law. Model trigonometric function through segments of the unit circle. Consider tangents and normals to a curve. Use a ruler to relate concavity and first and second derivatives.

Handle manipulatives which may be used to: reinforce $f(x)$ and consider functions of the form $y = af(x-h) + k$ and a possible inverse. Solve equations and systems. Discuss and compare functions. Explain and sketch rational and envelope functions.

3:55 - 4:10 Mellon Lecture Hall

"Discovery Textbook with Extensive Precalculus Review"

*David A. Cohen, Union County College*

The great majority of failing first-year calculus students falter in precalculus mathematics and not in new concepts presented. Textbooks already oversized with Analytic Geometry, Differential Equations, and applications problems now endeaver to include new problems for symbolic manipulation software. Certainly publishers cannot be expected to include precalculus concepts in these "unabridged" Calculus books. The need exists for a new generation of first year Calculus textbooks. Such a text exists, and sample chapters will be distributed.

This presentation will demonstrate how a specially keyed system provides review material to the student as needed. The Calculus component is presented in discovery style, rather than feeding students formulae that will soon be forgotten and never understood.

Institutions represented at the conference that have a course that integrates precalculus review with the first course in calculus:

Amherst College
Bates College
Dickinson College
Illinois Benedictine College
Moravian College
Mount Holyoke College
Saint Olaf College
SUNY Stonybrook
Suffolk University, Mass.
U.S. Coast Guard Academy
University of Massachussetts at Lowell
Western New England College

Other institutions that have a course that integrates precalculus review with the first course in calculus; these were interested in the conference, but unable to send a representative:

Dartmouth College
Saint Bonaventure University, NY
Seattle Central Community College

Institutions represented at the conference who plan in the academic year 1993-94 to offer a course that integrates precalculus review with the first course in calculus :

University of Hartford
University of Connecticut, Storrs
Ohio State University

If you know of other institutions that offer "calculus with review" courses (there are several), or plan to offer such a course in the next academic year, please convey that information to:

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# Appendix 4

# Reprint

"Integrating Precalculus Review with the First Course in Calculus,"
Alicia Sevilla and Kay Somers,
PRIMUS, vol. III, no. 1, 1993, pp. 35–41.

# INTEGRATING PRECALCULUS REVIEW WITH THE FIRST COURSE IN CALCULUS*

Alicia Sevilla and Kay Somers

ADDRESS: Mathematics Department, Moravian College, Bethlehem, PA 18018-6650 USA.

ABSTRACT: Moravian College offers a two-semester course that integrates precalculus, problem-solving techniques, and calculus. The course, including reasons for the integrated course and some positive outcomes, and the materials developed for it are discussed.

KEYWORDS: Precalculus, calculus, problem-solving.

In 1988 the Mathematics Department at Moravian College introduced a two-semester course that integrates precalculus topics as needed in a first calculus course. This course replaced the two-semester Precalculus-Calculus I sequence. Since the Fall of 1991, some members from the Mathematics Department at Moravian College and Northampton Community College have been writing materials for the course. This work is funded by a FIPSE (Fund for the Improvement of Post-Secondary Education, U.S. Department of Education) grant. In the first section of this article we include a brief description of the course, the reasons for change and some positive outcomes; in the second section we describe the materials that we are developing.

## 1. THE NEW COURSES: CALCULUS I WITH PRECALCULUS REVIEW PARTS I AND II

Every year approximately 60% of the freshman class intends to pursue a major that requires at least one semester of calculus. Based on our placement test and the students' high school records, we find that roughly half of these students are not adequately prepared to take calculus. Until 1988 these underprepared students took a standard precalculus course during their first

*Based on a presentation given at the Conference on the Teaching of Calculus held at Harvard University on 12-13 June 1992.



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semester, followed by one of these: a standard one semester Calculus I, which is the first of the three-semester calculus sequence; or a one-semester Applied Calculus terminal course, designed mainly for students of business and biology.

Some of the basic problems we found with the precalculus-calculus sequence were the following:

- Students lacked motivation in the precalculus course. They had seen the same material in high school, and the promise that they needed it for calculus next semester was not enough to motivate them to learn it better.
- The retention of material needed for calculus the next semester was extremely low.
- The morale of both students and teachers in the precalculus course was low.
- Many students took precalculus to satisfy a liberal education mathematics requirement and did not continue into calculus.
- Students who completed Applied Calculus did not have the prerequisite for Calculus II.

The new sequence, introduced in the Fall of 1988, consists of two one-semester courses: Calculus I with Precalculus Review Part I, and Calculus I with Precalculus Review Part II. These two courses together cover the same calculus material as our standard Calculus I plus review topics in algebra, elementary functions, and problem-solving. These review topics are introduced when needed for the calculus topics. The calculus topics covered in the first semester course are limits, continuity, and derivatives, including rules of differentiation for algebraic functions. The calculus topics included in the second semester course are derivatives of trigonometric, exponential and logarithmic functions, implicit differentiation, maximum and minimum values, curve sketching, applications of the derivative, area and the definite integral, and the Fundamental Theorem of Calculus. The review topics are intertwined with these calculus topics throughout the course.

To facilitate the transition to Calculus II for those students who continue the calculus beyond their first year, we use the same calculus textbook (Stewart, [2]) that we use for the standard sequence Calculus I through Calculus III. For the review topics we used a different text each year during the first three years and this past year we used draft materials prepared by several of those teaching the course.



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Since each year we have four or more sections of this sequence, every permanent member of our department has taught this course already. Those teaching the course at any given time, meet regularly throughout the term to discuss their sections' progress and to share materials and ideas about their teaching. This helps new instructors incorporate the precalculus material.

Mainly because of the lack of a suitable text to supplement a calculus textbook, we feel that our course has been still in preparation until now. In spite of this, we have observed some positive outcomes:

- The continuance rate from the first to the second semester has increased. During the two years 1986-87 and 1987-88, 65% of the students who passed precalculus the first semester continued to Calculus I or Applied Calculus in the second semester. During the four years of the new course since 1988, 82% of the students who passed Calculus I with Review Part I have continued to Part II the next semester.

- The success rate in completing Calculus I in one year has improved; for the two years 1986-87 and 1987-88, 40% of the students who enrolled in Precalculus successfully completed Calculus I or Applied Calculus the same year. During the last four years, 50% of the students who enrolled in Calculus with Review Part I successfully completed Parts I and II the same year.

- The new integrated course has been especially successful with adult students who have not studied mathematics for a long span of years but are highly motivated. This past year, 94% of the adult students who began Calculus I with Review Part I successfully completed the full-year course.

- Transition into Calculus II has been no problem; students who have taken Calculus I with Review have been well-prepared for Calculus II. In fact, we have had a few students who started in Calculus I with Review and continued to work for a computer science or mathematics major quite successfully.

- Some students' attitudes towards mathematics have improved.

- Students who pass Calculus I with Review Part I and choose not to continue to Part II learn something beyond the material they had in high school.

## 2. THE MATERIALS

In our search for materials for the course Calculus I with Review Parts I and II, we found that there were no supplemental materials available that were designed with the intention of integrating review of precalculus material in a slower-paced calculus course. This led us to develop a text designed for this purpose. A revised version of our materials is currently being used at Moravian College and Northampton Community College. Plans for publication have not been finalized.

There are three main goals for the supplemental text that we are developing:

1. Review the precalculus concepts as needed for calculus.

2. Motivate the students to study mathematics in general and calculus in particular with applied problems from their sphere of experience.

3. Provide exercises and examples which break some more complex ideas down into smaller, more manageable parts.

Our text, A Companion to Calculus [1], begins with an introduction that describes in general terms what calculus is, the fundamental role of functions in the study of calculus, and the use of symbols in mathematics. It is stressed that the language of mathematics includes four modes: words, pictures, numbers, and symbolic formulations, and that students need to be able to communicate in these four modes and be able to move from one mode to another. The introduction also contains exercises so that students can practice these transitions.

The first goal of the materials is to review precalculus concepts as needed for calculus. To accomplish this goal, the basic topics of Cartesian coordinates and functions are covered in Chapters 1 and 2, respectively. Absolute value inequalities and methods of algebraic simplification are needed to understand the definition of limit and to calculate limits, so they are reviewed in Chapter 3, Companion to Limits. Inequalities involving quadratic functions are needed in the study of domain and continuity of functions, so these inequalities are reviewed in Chapter 4, Companion to Continuous Functions.

A second goal of the materials is to motivate students. A map of Washington, DC is used to motivate a discussion of Cartesian coordinates, lines, and distance in Chapter 1. The two examples that follow appear in Chapter 5, The Role of Infinity.

## Example 5.1.

The potency (in milligrams) of vitamin C tablets is a function of the time t they have been stored. The graph in Figure 1 shows this relationship. Explain what the graph shows. Does the graph seem reasonable?

**Solution:** The graph shows that the tablets have an initial potency of 250 milligrams and potency remains constant at 250 milligrams until time $t = t_1$. From time $t_1$ to time $t_2$, the potency declines to 50 milligrams in a linear manner. After time $t_2$, the potency continues to decrease, but at a slower pace than before. The potency is actually never 0, but it is small for large values of $t$. The larger $t$ is, the closer the value of $P$ is to 0. In other words, storing the vitamins for a long time results in tablets with very little potency.

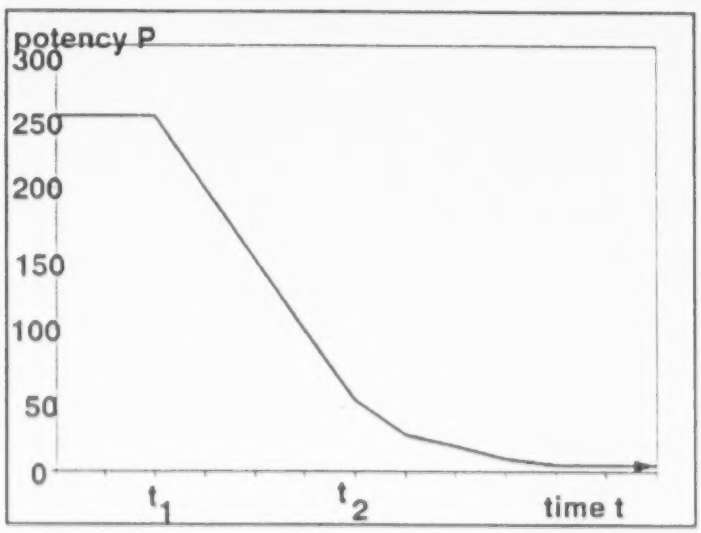


Figure 1. Potency of a vitamin C tablet.

## Example 5.2:

The number of items a typical assembly line worker can process in a day is a function of the number of days he or she has been working on the line. The graph in Figure 2 shows, on the vertical axis, the number of items processed, and on the horizontal axis, the number of days on the job.

a. The point (0,100) is on the graph. Interpret this.

b. Describe in words what happens as $t$ increases.

c. When can a worker be considered experienced?

d. What is the equation of the horizontal asymptote for this function?

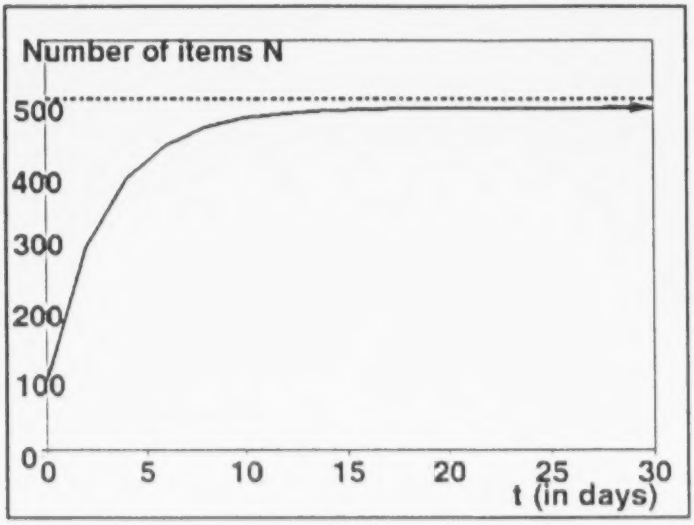


Figure 2. Number of items processed in a day.

Each example in the Companion is followed by its solution; at the end of each section there are similar exercises for students to work.

Chapter 6, Rates of Change, provides a thorough presentation of the concept of rates and average rate of change of a function before the introduction of the concept of the derivative, which is an instantaneous rate of change.

Chapter 7, Companion to Rules of Differentiation, contains a section on decomposition of functions which prepares students to recognize when it is appropriate to use the various differentiation rules. This chapter is the final chapter of the materials used in the first term of the course; the second term begins with a review of Trigonometry.

A third goal of the materials is to provide exercises and examples which break some more complex ideas down into smaller, more understandable parts. The section on decomposition of functions mentioned previously is one illustration of this. The following example from Chapter 11, Companion to Related Rates, also demonstrates this idea.

### Example 11.2:

For each of the following statements, draw a picture, assign a letter name to each quantity that is changing over time, and interpret the statement involving rate of change in terms of symbols.

a. A man leaves his house and walks directly north at a rate of 4 miles per hour.

b. Water is being pumped out of a large storage tank at a rate of 5 cubic feet per minute.

c. One hour after it leaves the station, a train is traveling at 70 miles per hour.

The solution to this exercise presented in the text contains words, pictures, numbers, and symbols.

Chapter 16, Companion to Extreme Values of a Function, contains sections on solving equations, determining constraints, and translating descriptive statements into algebraic notation. Problems are broken down into component parts so students can first focus on each part, and then tackle the problems found in a typical calculus text.

At the end of the two-semester sequence of Calculus I with Review, Parts I and II, students have covered all the material from a one-semester regular calculus course and should be prepared to continue with Calculus II, if they choose to do so.

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2. Stewart, J. 1991. *Calculus, Early Transcendentals. Second Edition*. Pacific Grove CA: Brooks/Cole Publishing Company.

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PRIMUS March 1993 Volume III Number 1

# SHORT ESSAY TOPICS FOR CALCULUS*

M. L. Platt

ADDRESS: Department of Mathematics, Salem State College, Salem MA 01970 USA.

ABSTRACT: Short essay questions have been introduced into the calculus course as one technique to involve students with their own learning. This paper presents some of the ways this has been done.

KEYWORDS: Teaching methods, writing.

## INTRODUCTION

Student research projects can be used as an alternative to the "lecture and listen" approach to teaching. They also help to involve the students with their own learning, as suggested in Everybody Counts [1]. Student research projects in calculus have had many positive and rewarding benefits. They have fostered a mixture of independent thinking and cooperative learning. They provide an opportunity to work on multistep problems using a mixture of guided self-study and collaborative discovery. However, along with the good there has been some bad. This paper attempts to explore these issues and propose suggestions for ways to address these issues.

## REPORT INSTRUCTIONS

Research projects were first incorporated into the calculus sequence at Salem State College during the Fall semester of 1991. The directions attached to the projects were adopted from material in [2]:

### INSTRUCTIONS FOR WRITING THE REPORT

The report is to be a thoughtful and neatly organized document that

*Based on a presentation given at the Conference on the Teaching of Calculus held at Harvard University on 12-13 June 1992.

# Appendix 5

## Preprint

"A Report on a Project to Develop Course Materials to Integrate Precalculus Review with the First Course in Calculus," Alicia Sevilla, Kay Somers, and Doris Schattschneider, Proceedings of the Conference on Calculus and Precalculus Reform (Monticello, Illinois, April 1993), to be published by the Mathematical Association of America, Fall 1993.

To be published in the Proceedings of the Conference on Calculus and Precalculus Reform, MAA, 1993

# A Report on a Project to Develop Course Materials to Integrate Precalculus Review with the First Course in Calculus

Alicia Sevilla, Kay Somers, and Doris Schattschneider
Moravian College
Bethlehem, PA 18018-6650

In 1988 the Mathematics Department at Moravian College introduced a two-semester course that intertwines precalculus topics with the material in a first calculus course. This course replaced a traditional two-semester Precalculus-Calculus I sequence. Since the summer of 1991, some members from the Mathematics Departments at Moravian College and Northampton Community College have written materials for the course. This work has been funded by a 2-year FIPSE (Fund for the Improvement of Post-Secondary Education, U.S. Department of Education) grant.

## The Course

The new sequence, introduced in the Fall of 1988, consists of two one-semester courses: Calculus I with Precalculus Review Part I, and Calculus I with Precalculus Review Part II. These two courses together cover the same calculus material as a standard Calculus I course, plus review topics in algebra, elementary functions, and problem-solving. These review topics are introduced when needed for the calculus topics and put into the context in which they will be used to solve calculus problems. The calculus topics covered in the first semester course are limits, continuity, and derivatives (including rules of differentiation and implicit differentiation) for algebraic functions. The calculus topics included in the second semester course are derivatives of trigonometric, exponential and logarithmic functions, extreme values, curve sketching, other applications of the

derivative, antiderivatives, area and the definite integral, and the Fundamental Theorem of Calculus. The review topics are integrated with these calculus topics throughout the two semesters.

To facilitate the transition to Calculus II for those students who continue the calculus beyond their first year, we use the same calculus textbook that we use for the standard 3-semester sequence, Calculus I, II, III. When we first prepared the new course in 1988, we found that there were no supplemental materials available that were designed with the intention of integrating review of precalculus material in a slower-paced calculus course. For the first three years in which the course was taught, we used different College Algebra review texts to provide the supplemental material but found these texts were not well suited to our purpose.

This led to our proposal to FIPSE to develop suitable materials ourselves. There are three main goals for the supplemental text that we have written: (1) Review the precalculus concepts *as needed* for calculus. (2) Motivate the students to study mathematics in general and calculus in particular with applied problems from their sphere of experience. (3) Provide exercises and examples that break complex ideas into smaller, more manageable parts.

The Text

Our text, *A Companion to Calculus*, begins with an introduction that describes in general terms what calculus is, the fundamental role of functions in the study of calculus, and the use of symbols in mathematics. It is stressed that the language of mathematics includes four modes: words, pictures, numbers, and symbolic formulations, and that students need to be able to communicate in these four modes and be able to move from one mode to another. The introduction also contains exercises so that students can

practice these transitions. Throughout the text we aim to present each topic in as many of the modes as possible.

The first goal of the materials is to review precalculus concepts as needed for calculus. The basic topics of Cartesian coordinates and functions are covered in Chapters 1 and 2. Different ways to represent functions using the four modes of the language of mathematics are stressed. Simplification of algebraic expressions and the solutions of linear and quadratic equations and inequalities are reviewed in several different chapters. For example, linear and absolute value inequalities are discussed in Chapter 3, *Companion to Limits*. Inequalities that involve quadratic functions are reviewed in Chapter 4, *Companion to Continuous Functions*, and these and more general inequalities are examined again in Chapter 16, *Companion to Extreme Values of a Function*. Methods to find or to approximate zeros of polynomials are also reviewed in these chapters.

Chapter 6, *Rates of Change*, provides a thorough presentation of the concepts of rate and average rate of change of a function before the introduction of the concept of derivative, which is an instantaneous rate of change. In Chapter 9, *Companion to Implicit Differentiation*, examples show how to solve an equation for $\frac{dy}{dx}$.

Rules of exponents for rational exponents are discussed first in Chapter 7, *Companion to Rules of Differentiation and the Chain Rule* and are reviewed and extended to real exponents in Chapter 13, *Companion to Exponential Functions*. Chapter 7 also contains a section on decomposition of functions which prepares students to recognize when it is appropriate to use the various differentiation rules. This topic is addressed again in Chapter 18, *Companion to Antidifferentiation*.

In addition to reviewing precalculus topics as needed for calculus, the materials are designed to reinforce ideas from earlier in the course. The

following exercise in Chapter 13, *Companion to Exponential Functions*, relates exponential functions to functions discussed in Chapter 2.

Exercise: Match each of the following functions to its graph below.

a. $f(x)=2^x$ b. $g(x)=x^2$ c. $h(x)=(1/2)^x$ d. $F(x)=x^{1/2}$

e. $G(x)=e^2$ f. $H(x)=$

The statement of the problem is followed by six graphs.

A second goal of the materials is to motivate students to study mathematics, and in particular, calculus. In Chapter 1 a map of Washington, DC, is used to motivate a discussion of the use of Cartesian coordinates and the distance formula. In Chapter 2, *Functions*, step functions are illustrated with the following example.

**Example 2.11**: To make an operator-assisted telephone call to London, the Phone Company charges \$5.50 for the first three minutes and \$.75 for any additional minute (or fraction of a minute). Draw a graph and also describe in symbols the function that represents the charges (in dollars) as a function of the length of the call (in minutes). How much does it cost to call for 10 minutes? 8-1/2 minutes?

This example, as with each example in the *Companion*, is followed by a full solution. At the end of each section there are similar exercises for students to work. The following exercise appears in Chapter 2 as a follow-up to the example above.

Exercise: The following table indicates the dose of a medication a child should receive according to the child's weight:

| Weight (in pounds) | Dose (teaspoons) |
|---|---|
| 24-35 | 1 |
| 36-47 | 1-1/2 |
| 48-59 | 2 |
| 60-71 | 2-1/2 |
| 72-95 | 3 |

Let *f* be the function that assigns to each weight the corresponding dose.

a. Draw a graph of *f*.

b. Describe *f* in symbols using function notation.

c. What is the dose for a child that weighs 46 lbs.?

Here is an example from Chapter 5, *The Role of Infinity*. It illustrates the four modes of communication stressed throughout the materials -- words, pictures, numbers, and symbols.

**Example 5.1**: The potency (in milligrams) of vitamin C tablets is a function of the time t they have been stored. The graph in Figure 5.1 shows this relationship. Explain what the graph shows. Does the graph seem reasonable?

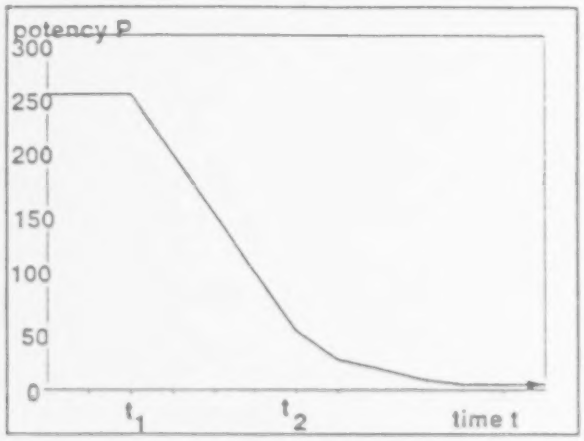


Figure 5.1

**Solution**: The graph shows that the tablets have an initial potency of 250 milligrams and potency remains constant at 250 milligrams until time $t=t_1$. From time $t_1$ to time $t_2$, the potency declines to 50 milligrams in a linear manner. After time $t_2$, the potency continues to decrease, but at a slower pace than before. The potency is actually never 0, but it is small for large values of t. The larger t is, the closer the value of P is to 0. In other words, storing the vitamins for a long time results in tablets with very little potency.

A third goal of the materials is to provide examples and exercises which break down complex ideas into smaller, more understandable parts. The following examples, the first from Chapter 11, *Companion to Related Rates*, and the second from Chapter 16, *Companion to Extreme Values of a Function*, illustrate how to set up equations.

**Example 11.1**: A radio tower has height 130 feet and has been assembled on the ground, lying on its side. A motorized device raises the tower until it is in its vertical position.

a. Draw and label a picture to represent this situation, and identify any quantities that change as time changes.

b. Find a relationship between the variable quantities and express the relationship as an equation.

**Example 16.5**: A farmer wishes to fence a rectangular field to enclose an area of 9,000 square feet. The south and west sides of the fence will cost \$3.80 per foot and the other two sides will cost \$4.05 per foot. The farmer

wants to know what dimensions of the field will minimize his cost. Give the function and its domain that he should consider in order to find the answer. (Assume that the farmer's land is a square, 1 mile on each side.)

Evaluation of the Course and Text

We tested a draft version of *A Companion to Calculus* in all four sections of Calculus I with Review (over 100 students) during the 1991-92 academic year. To assess the effect of this course and our materials, we carried out a four-part evaluation. A full evaluation report for the 1991-92 year is available by request; a similar report for 1992-93 will be completed in summer 1993.

First, we used a survey to evaluate student attitudes toward learning mathematics, perception of what was important in the course, opinions about the *Companion* material, the calculus text and the course. This survey was administered to all the students in the course at the end of each semester.

Second, final examinations were designed in cooperation with the instructors of the four sections of the standard one-semester Calculus I. To compare the performance of students in this course with that of students in the one-year Calculus with Review course, some of the questions on the standard Calculus I final exam appeared on the final exam of Calculus I with Review Part I and others appeared in the final exam of Calculus I with Review Part II.

Third, to measure the improvement in understanding the review material, the final examination of Calculus with Review Part I also contained questions from the departmental placement test administered to all freshmen before registration for the course.

Fourth, data on course enrollment, attrition, and completion of the Calculus with Review course was compiled for all years beginning with 1988

and compared with similar data for the Precalculus-Calculus I sequence for the years 1986-1988.

The results of the evaluations obtained in the Fall Term and Spring Term gave us some hopeful signs of success. On questions that were common to the final exams in the "regular" Calculus I and the Calculus I with Review, students in the first course did only slightly better than those in Calculus I with Review. In the Fall exam, evening section (adult) students in the latter course actually outscored those in the "regular" course on 3 (out of 12) questions, and in the Spring exam the mean score for the Calculus with Review students was significantly higher than that of the Calculus I students on one of the 12 common questions.

The comparison of individual student scores on questions on their placement examinations and scores on the same questions on the final examination for the Calculus I with Review course showed that most improved their scores. A comparison of mean scores for common questions on the two tests showed improvement in every case.

The data on attrition and course completion has also been encouraging. The continuance rate from the first to the second semester has increased. During the two years 1986-87 and 1987-88, only 65% of the students who passed Precalculus the first semester continued to Calculus I or Applied Calculus in the second semester. During the four years of the new course, 1988-92, 81% of the students who passed Calculus I with Review Part I have continued to Part II the next semester. The success rate in completing Calculus I in one year has improved. For the two years 1986-87 and 1987-88, 40% of the students who enrolled in Precalculus successfully completed Calculus I or Applied Calculus the same year. During the last four years, 50% of the students who enrolled in Calculus with Review Part I successfully completed Parts I and II the same year.

The results of the student attitude survey have also been encouraging. Most students indicated the *Companion* text was helpful, and preferred it to the calculus text. Most indicated an improvement in their ability to solve mathematical problems and felt the material could be useful in their major.

Other favorable outcomes of the course have been noted. Transition into Calculus II has been no problem; in fact, we have had a few students who started in Calculus I with Review and continued to work for a computer science or mathematics major quite successfully. Students who pass Calculus I with Review Part I and choose not to continue to Part II learn something beyond the material they had in high school, unlike when they took precalculus and did not continue on to calculus.

The *Companion* has been especially successful with adult students who have not studied mathematics for a long span of years but are highly motivated. This past year, 28 of the 29 adult students who began Calculus I with Review Part I successfully completed the full-year course.

On June 18-19, 1993, a dissemination conference at Moravian College will cap the FIPSE-funded project. Many other institutions have shown interest in this integrated approach to teaching calculus to under-prepared students; we have learned of several colleges that have developed their own integrated 1-year course. It is our hope that a version of our *Companion* will be published commercially in the near future. In the meantime, we welcome requests for this text for class-testing.